An Updated User's Guide to SOFR

The Alternative Reference Rates Committee

February 2021

Executive Summary

This note is intended to help explain how market participants can use SOFR in cash products. The ARRC has stated that those who are able to use SOFR should not wait for forward-looking term rates in order to transition, and the note lays out a number of considerations that market participants interested in using SOFR will need to consider:

- Financial products either explicitly or implicitly use some kind of *average* of SOFR, not a single day's reading of the rate, in determining the floating-rate payments that are to be paid or received. An average of SOFR will accurately reflect movements in interest rates over a given period of time and smooth out any idiosyncratic, day-to-day fluctuations in market rates.

- Issuers and lenders will face a technical choice between using a *simple* or a *compound* average of SOFR as they seek to use SOFR in cash products. In the short-term, using simple interest conventions may be easier since many systems are already set up to accommodate it. However, compounded interest would more accurately reflect the time value of money, which becomes a more important consideration as interest rates rise, and it can allow for more accurate hedging and better market functioning.

- Users need to determine the period of time over which the daily SOFRs are observed and averaged. An *in advance* structure would reference an average of SOFR observed before the current interest period begins, while an *in arrears* structure would reference an average of SOFR over the current interest period.

- SOFR *in advance* is operationally easier to implement, but SOFR in arrears will reflect movements in rates contemporaneously. An average of SOFR *in arrears* will reflect what *actually happens* to interest rates over the period; however it provides very little notice before payment is due. There have been a number of conventions designed to allow for a longer notice of payment within the *in arrears* framework. These include payment delays, lookbacks, and lockouts, and, as described in the note, different markets have successfully adopted each of these. The note also discusses conventions for *in advance* payment structures and *hybrid* models that can substantially reduce the basis relative to *in arrears* while still providing borrowers the same length of notice that they have with LIBOR.

The note also explains the interaction between SOFR and the type of forward-looking term rates that the ARRC has set a goal of seeing produced once SOFR derivative markets develop sufficient depth. While these term rates can be a useful tool for some and an integral part of the new ecosystem, hedging these rates will also tend to entail more costs than using SOFR directly and their use must be consistent with the functioning of the overall financial system. For this reason, the ARRC sees some specific productive uses for a forward-looking SOFR term rate, in particular as a fallback for legacy cash products referencing LIBOR and in loans where the borrowers otherwise have difficulty adapting to the new environment.

Background

In 2014, the Federal Reserve convened the Alternative Reference Rates Committee (ARRC) and tasked the group with identifying an alternative to U.S. dollar LIBOR that was a robust, IOSCO-compliant, transaction-based rate derived from a deep and liquid market. In 2017, the ARRC fulfilled this mandate by selecting the Secured Overnight Financing Rate, or SOFR. SOFR is based on overnight transactions in the U.S. dollar Treasury repo market, the largest rates market at a given maturity in the world. National working groups in other jurisdictions have similarly identified overnight nearly risk-free rates (RFRs) like SOFR as their preferred alternatives.

SOFR has a number of characteristics that LIBOR and other similar rates based on wholesale term unsecured funding markets do not:

- It is a rate produced by the Federal Reserve Bank of New York for the public good;
- It is derived from an active and well-defined market with sufficient depth to make it extraordinarily difficult to ever manipulate or influence;
- It is produced in a transparent, direct manner and is based on observable transactions, rather than being dependent on estimates, like LIBOR, or derived through models; and
- It is derived from a market that was able to weather the global financial crisis and that the ARRC credibly believes will remain active enough in order that it can reliably be produced in a wide range of market conditions.

However, SOFR is also new, and many are unfamiliar with how to use it. SOFR is also an overnight rate, and while the ARRC believes that most market participants can adapt to this by using compound or simple averaging over the relevant term, the ARRC has at the same time set a goal of seeing an administrator produce a forward-looking term rate based on SOFR derivatives (once these markets develop to sufficient depth) in order to aid those cash market participants who may have greater difficulty in adapting to an overnight rate.

The national working groups in the other currency jurisdictions each independently reached the same conclusion that there were no viable robust term rate alternatives to LIBOR. Like the ARRC, each has chosen either an unsecured or secured overnight rate, depending on the characteristics of their national markets (see Table 1).[1]

Table 1: Selected RFRs

U.S. Dollar	SOFR	Overnight secured repo rate
Sterling	SONIA	Overnight unsecured rate
Japanese Yen	TONA	Overnight unsecured rate
Euro	ESTER	Overnight unsecured rate
Swiss Franc	SARON	Overnight secured repo rate

[1] Further information on the work of each of the national working groups in other currency jurisdictions can be found in the FSB's Progress Report on Reforming Major Interest Rate Benchmarks, October 2017.

This note is intended to help explain how market participants can use SOFR in cash products and to explain the forward-looking term rates the ARRC seeks to see published in the future and where the ARRC believes those rates can be most productively used. The term rates can be a useful tool for some and an integral part of the new ecosystem; but their use also needs to be consistent with the functioning of the overall financial system. In particular, those who are able to use SOFR should not wait for the term rates in order to transition.[2] The LIBOR transition will be challenging, and it is not in the interest of market participants to put off taking action nor can the ARRC guarantee that an administrator can produce a robust, IOSCO-compliant forward-looking term rate before LIBOR stops publication. The ARRC sees some specific uses, in particular as a fallback for legacy cash products referencing LIBOR and in loans where the borrowers otherwise have difficulty in adapting to the new environment, where the term rates can be most productively used. For many other purposes, the ARRC believes it should be possible to use compound or simple averages of SOFR and that many users will come to find it more convenient to do so once they become more familiar with the new environment.

A. SOFR

SOFR is published on a daily basis by the Federal Reserve Bank of New York (FRBNY), in cooperation with the Office of Financial Research, and reflects the cost of overnight borrowing and lending in the U.S. Treasury repo market. Borrowing in this market reflects the best measure of the private sector risk-free rate, because it is collateralized with U.S. Treasury securities, which the lender returns once the borrower returns the cash borrowed. SOFR is a fully transactions-based rate and has the widest coverage of any Treasury repo rate available, incorporating tri-party repo data, the Fixed Income Clearing Corporation's (FICC) GCF Repo data, and bilateral Treasury repo transactions cleared through FICC.[3] Throughout 2020, the average daily volume of transactions underlying SOFR was close to $1 trillion, representing the largest rates market at any given tenor in the United States.

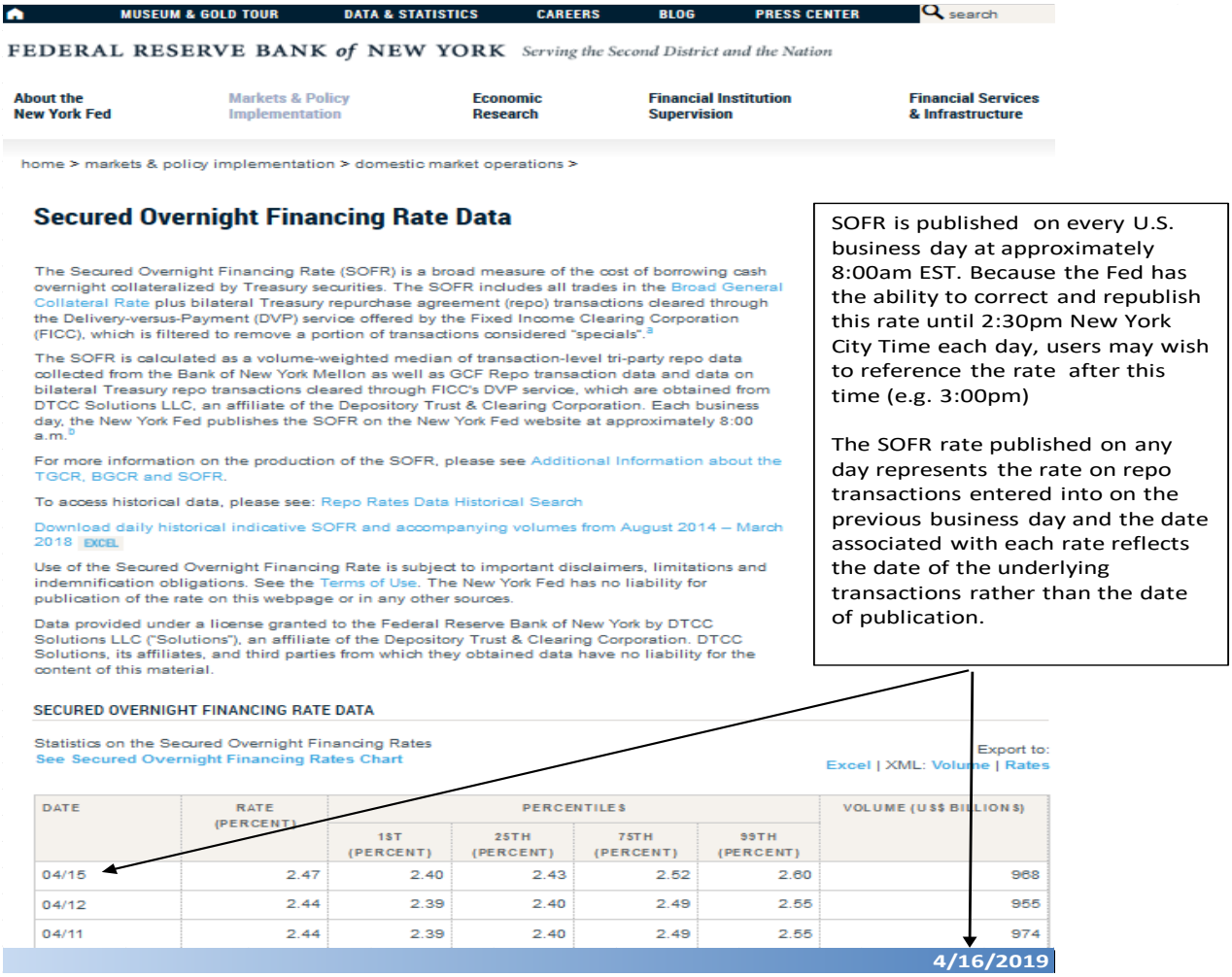
Because of its range of coverage, SOFR is a good representation of general funding conditions in the overnight Treasury repo market. As such, it reflects an economic cost of lending and borrowing relevant to the wide array of market participants active in these markets, including not only broker-dealers, but also money market funds, asset managers, insurance companies, securities lenders, and pension funds. SOFR moves closely with other available repo rates and has tended to lie in the middle of the range between other available repo rates. SOFR is generally a few basis points higher than rates based only on tri-party transactions (such as the Bank of New York Mellon's Treasury Tri-Party Repo Index or the tri-party general collateral rate produced by FRBNY) but is generally lower and less volatile than DTCC's Treasury GCF Repo Index.

SOFR is calculated as a volume-weighted median of transaction-level data observed over the course of a business day and is published on the FRBNY website at approximately 8:00 a.m. ET on the next business day (see the accompanying figure). Looked at another way, SOFR is published on the day that the overnight repo transaction is to be repaid rather than on the day that the transaction is entered into. This publication schedule is due to the need to receive and fully vet the large amounts of data

[2] The FSB has recognized that there may be a role for these types of forward-looking term rates, but the FSB has also stated that it considers that the greater robustness of overnight rates like SOFR makes them a more suitable alternative than these forward-looking term rates in the bulk of cases.

[3] Further details on the structure of the U.S. Treasury repo market is available in the ARRC's Second Report; see also Bowman, Louria, McCormick and Styczynski (2017).

underlying SOFR before the rate is published. SOFR is published for the business days that the Treasury repo market is open on, which are generally U.S. government securities secondary-market trading days as determined by SIFMA[4]



Although SOFR is published at about 8:00 a.m. ET, if any errors are subsequently discovered in the transaction data in the calculation process that underlies it, or if any missing data subsequently became available, then SOFR may be republished on the same day. In such cases, the affected rate may be republished at approximately 2:30 p.m. ET. Rate revisions will only be effected on the same day as initial publication and will only be republished if the change in the rate exceeds one basis point. To date, there have been no rate republications for SOFR, but if at any time a rate is revised, a footnote would indicate the revision.[5]

[4] SIFMA's calendar of government securities trading days can be found at https://www.sifma.org/resources/general/holiday-schedule/#US.

[5] Although SOFR has not been republished, on May 31, 2019, it was published based on FRBNY's contingency rate calculation methodology. This methodology involves the use of a highly detailed survey of Primary Dealer's repo borrowing activity conducted by FRBNY every day. More information and this event and a summary of FRBNY's data contingency procedures can be found on FRBNY's website.

In addition to producing SOFR, the Federal Reserve Bank of New York also publishes 30-day, 90-day, and 180-day averages of SOFR and a SOFR Index daily on its website.

SOFR averages can be used either *in advance* or *in arrears* (concepts discussed further below) depending on whether the averages are applied at the start or end of an interest period, but they are more likely to be used in advance. They are calculated based on ISDA's compound SOFR formula (described in the Appendix), although the calculations for the averages may start on a weekend or holiday in order to ensure that they cover a fixed number of days, and in that respect they differ from the standard convention in the SOFR OIS market which would always start and stop on a business day. To allow users to calculate a compound SOFR based on ISDA's definitions over any start and ending date, for example a monthly compound average based on the modified following business day convention, FRBNY also publishes the SOFR Index. The Index, and how it can be interpolated to calculate a compound average over a period that starts or stops on a nonbusiness day, is covered further in the Appendix.

SOFR and the SOFR averages and Index are available both on FRBNYs website and also through an API or through various data providers. Documentation for these rates can be found on the FRBNY website, including policies and procedures and detailed evidence of IOSCO compliance. In addition to SOFR, FRBNY produces a number of daily statistics, including the volume of transactions underlying SOFR and selected percentiles of the rates observed across transactions, to aid market participants in judging the quality of the rate.

Historical SOFR Data

FRBNY, in cooperation with the Office of Financial Research, began publishing SOFR on April 3, 2018, but there is a longer history of repo rate data based on several sources that have been made available by FRBNY. Prior to the start of official publication, FRBNY also released data from August 2014 to March 2018 representing modeled, pre-production estimates of SOFR that are based on the same basic underlying transaction data and methodology that now underlie the official publication. While the full set of data sources required to calculate SOFR did not exist prior to August 2014, FRBNY has also separately released a much longer historical data series based on primary dealers' overnight Treasury repo borrowing activity. Bowman (2017) provides evidence that this historical data should be a good proxy for how a rate like SOFR would have behaved over a longer period of time.

B. How Can Financial Products Use Overnight Rates?

Although many market participants have become accustomed to using term IBORs, they are a relatively new phenomenon, and financial markets were able to function perfectly well before these rates were widely adopted. There is in fact a long history of use of overnight rates in financial instruments. In the United States, futures referencing the effective federal funds rate (EFFR) have traded for more than 30 years and overnight index swaps (OIS) referencing EFFR have traded for almost 20 years. Banks in the United States also have a history of offering loans based on the Prime Rate, which is essentially an overnight rate, or overnight LIBOR, and there have been floating rate

notes ("FRN") issued based on the EFFR or, more recently, SOFR. Other countries have similar experiences; for example, in Canada, most floating-rate mortgages are based on overnight rates.

Averaged Overnight Rates

Many financial products have used overnight rates as benchmarks, but one key thing to keep in mind is that these financial products either explicitly or implicitly use some kind of *average* of the overnight rate, not a single day's reading of the rate, in determining the floating-rate payments that are to be paid or received.

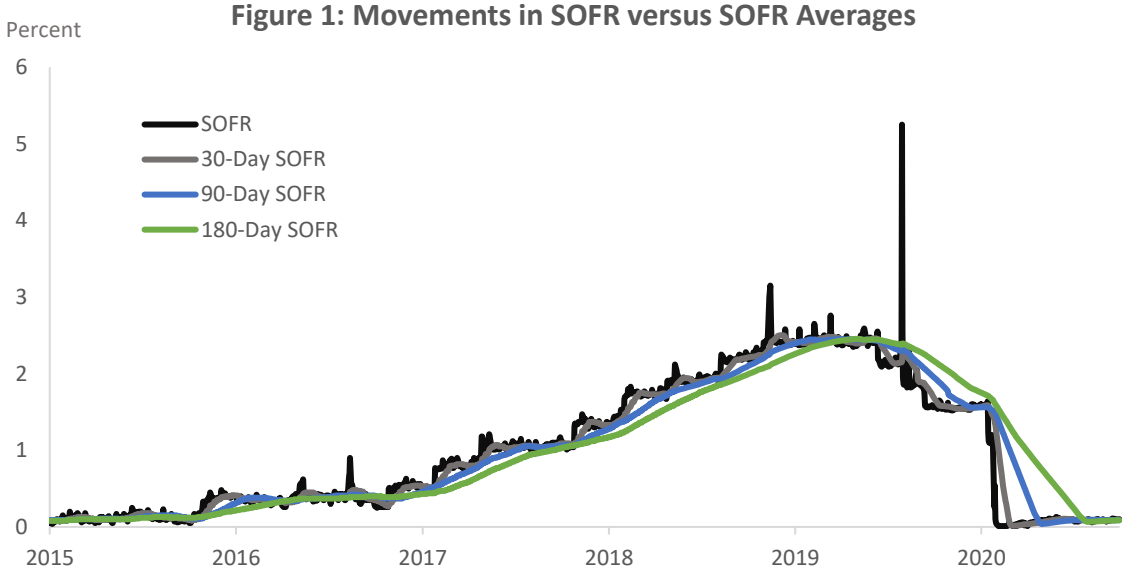
There are two essential reasons why financial products use an average of the overnight rate:

- First, an average of daily overnight rates will accurately reflect movements in interest rates over a given period of time. For example, SOFR futures and swaps contracts are constructed to allow users to hedge future interest rate movements over a fixed period of time, and an average of the daily overnight rates that occur over the period accomplishes this.

- Second, an average overnight rate smooths out idiosyncratic, day-to-day fluctuations in market rates, making it more appropriate for use.

This second point can be seen in Figure 1. On a daily basis, SOFR can exhibit some amount of idiosyncratic volatility, reflecting market conditions on any given day, and a number of news articles pointed to the jump in SOFR and other overnight repo rates in the fall of 2019. However, although people often focus on the type of day-to-day movements in overnight rates shown by the black line in the figure, it is important to keep in mind that the type of averages of SOFR that are referenced in financial contracts are much smoother than the movements in overnight SOFR.

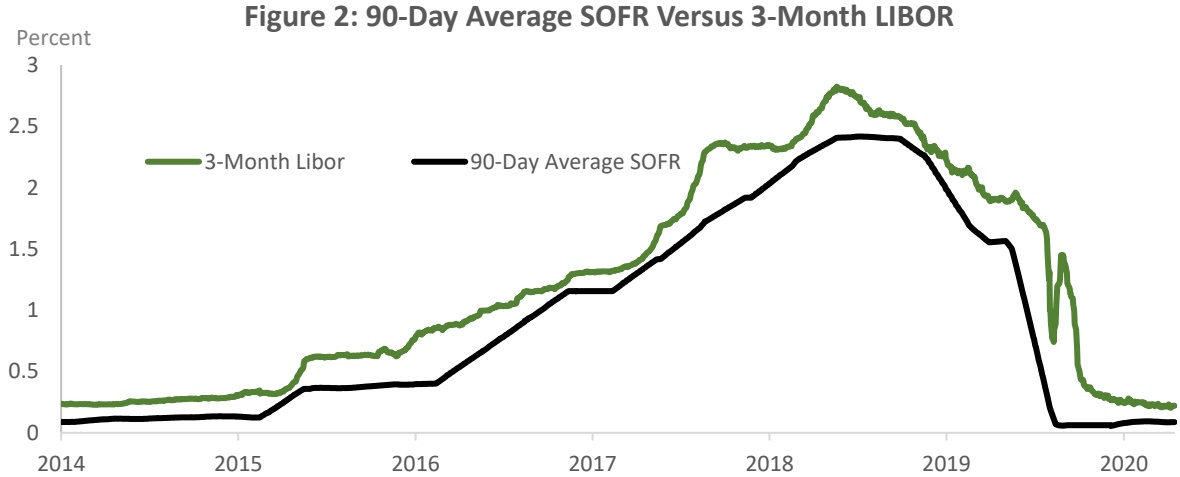
Figure 1: Movements in SOFR versus SOFR Averages



Source: Federal Reserve Bank of New York; Federal Reserve Board staff calculations. Data from August 2014 to March 2018 represent modeled, pre-production estimates of SOFR.

The amount of daily volatility in SOFR can change over time and depends on a number of factors, including the monetary policy framework and day-to-day fluctuations in supply and demand, but regardless of these factors, using an averaged overnight rate smooths out almost all of this type of volatility. As was emphasized in the ARRC's Second Report and is still the case today even over the year end, a three-month average of SOFR is less volatile than 3-month LIBOR (Figure 2). This was true even in 2019 over the period when overnight repo rates experienced their short spike in volatility.

Figure 2: 90-Day Average SOFR Versus 3-Month LIBOR



Source: Federal Reserve Bank of New York, ICE Benchmarks Administration; Federal Reserve Board staff calculations.
Data from August 2014 to March 2018 represent modeled, pre-production estimates of SOFR.

Compound versus Simple Averaging

Although financial products will all tend to use an averaged overnight rate, they may exhibit some technical differences in how these averages are calculated. The choice of a particular averaging convention need not affect the overall rate paid by the borrower, because the differences between them are generally small and other terms can be adjusted to equate the overall cost, but nonetheless issuers and lenders will face a technical choice between using a *simple* or a *compound* average as they seek to use SOFR in cash products. Since this is a source of confusion for some, we will explain both here.

Simple and compound averages reflect a technical difference in how interest is accrued by using either *simple* or *compound interest*. Financial markets participants have developed a number of conventions for calculating the amount of interest owed on a loan or financial instrument.[6] One area where this is the case is in the choice convention between simple versus compound interest:

- *Simple interest* is a long-standing convention, and in some respects is easier from an operational perspective. Under this convention, the additional amount of interest owed each day is calculated by applying the daily rate of interest to the principal borrowed, and the payment due at the end of the period is the sum of those amounts.

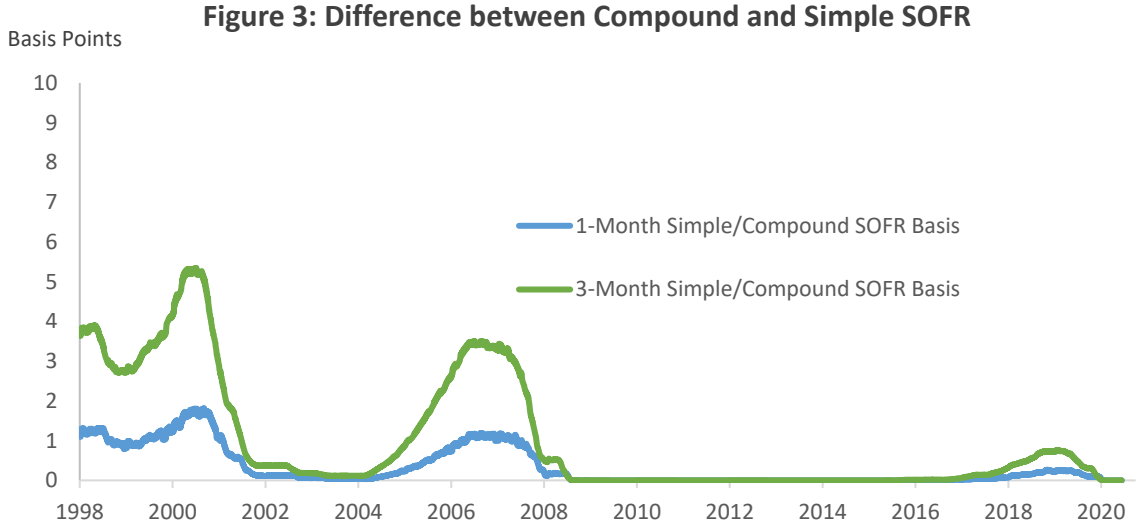
[6] Some of those conventions were developed before modern computing made such calculations routine, at a time when interest had to be calculated manually or by looking up the answer in tables. As computing has become widespread, new conventions have developed, but in many cases both older and newer conventions coexist in the market.

- *Compound interest* recognizes that the borrower does not pay back interest owed on a daily basis and it therefore keeps track of the accumulated interest owed but not yet paid. The additional amount of interest owed each day is calculated by applying the daily rate of interest both to the principal borrowed and the accumulated unpaid interest.

From an economic perspective, compound interest is the more correct convention. For example, if someone holds a bank account or money market fund paying overnight interest, then they receive compounded interest. OIS markets also use compound interest, and thus instruments that use compound interest will be easier to hedge. On the other hand, simple interest is easier to calculate and many systems are designed around its use, for example, in the United States loan and short-term FRN systems using overnight LIBOR or EFFR were built around the use of simple interest, and those systems would require investment to change in order to incorporate compound interest calculations.

Beyond the math, it is perhaps most important to understand that the difference between the two concepts is typically quite small at lower interest rates and over short periods of time. Any differences can also be accounted for by adjusting the rate or margin. Historically, the difference between simple and compounded interest on SOFR would have ranged between 0 and 6 basis points over the last two decades (Figure 3), with the difference being larger when rates moved higher or when the payment frequency was longer.

Figure 3: Difference between Compound and Simple SOFR



Source: Federal Reserve Bank of New York; Federal Reserve Board staff calculations. Data from August 2014 to March 2018 represent modeled, pre-production estimates of SOFR.

In the short-term, using SOFR with simple interest conventions may be easier since many loan and FRN systems are already set up to accommodate it. However, compounded interest would more accurately reflect the time value of money, which becomes a more important consideration as interest rates rise, and it can allow for more accurate hedging. Of course, the choice between compounded and simple interest is a decision between counterparties and would entail investments to update systems in order to accommodate a compounded rate. The ARRC has recognized that either convention can be used and that the choice will depend on the specifics of the product, including trading and other conventions that may interact with the choice of interest accrual.

Apart from the choice between simple and compound interest, there are a number of other conventions that need to be set, though they generally should have less economic impact on the

amount of interest payments. Amongst others, these include the choice of day count convention (which determines how annualized rates are quoted) and how the rate is to be applied over weekends and holidays (whether to use the rate on transactions taking place before the weekend or holiday, which mirrors how repo markets operate, or the rate after). The Appendix provides the formulation ISDA uses in its conventions and provides an example of the calculations behind compounded interest.

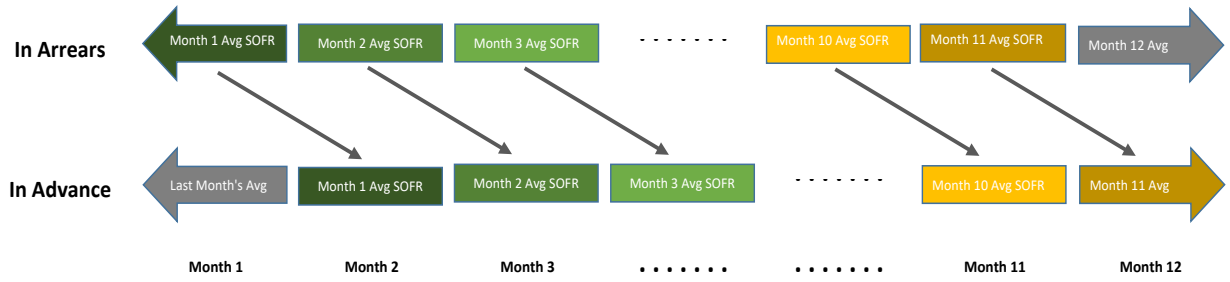
C. *Notice of Payment (In Arrears versus In Advance and In Advance Hybrids)*

Most of the contracts that reference LIBOR set the floating rate based on a value of LIBOR determined before the *beginning of the interest period*. This convention is termed *in advance* because the floating-rate payment due is set in advance of the start of the interest period. But not all LIBOR contracts take this form; some LIBOR swaps reference a value of LIBOR determined at the *end of the interest period*. This convention is termed *in arrears*.[7]

These conventions are used with overnight rates also. An *in advance* payment structure based on an overnight rate would reference an average of the overnight rates observed before the current interest period began, while an *in arrears* structure would reference an average of the rate over the current interest period. As noted above, an average overnight rate *in arrears* will reflect what *actually happens* to interest rates over the period and will therefore fully hedge interest rate risk in a way that LIBOR or a SOFR-based forward-looking term rate will not.

The tension in choosing between *in arrears* and *in advance* is that many borrowers will reasonably prefer to know their payments ahead of time – well ahead of time for some borrowers – and so prefer *in advance*, while investors will reasonably prefer returns based on rates over the interest period (i.e., *in arrears*) and will tend to view rates set *in advance* as "out of date." Nonetheless, there is actually a tight economic link between the two conventions – absent any changes in balance, the payments made on a SOFR in advance loan or security are equal to the payments that would be made using SOFR in arrears, but those payments are lagged by one interest period as shown in the next figure.

Figure 4: Comparing Payments on a 12-Month Floating-Rate Loan Using SOFR in Arrears versus in Advance

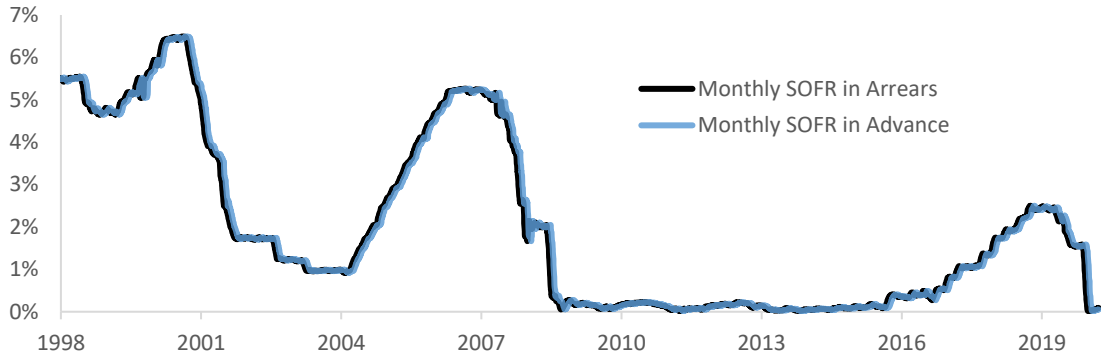


As a result, the two types of SOFR averages have moved closely together over time, as shown in Figure 5. In terms of hedging general interest rate risk, both structures for using SOFR will reflect

[7] Although this convention doesn't necessarily have to imply that payment is made after the interest period has concluded, payment will frequently be made 1-2 days after the period has ended and in that sense is in arrears relative to the end of the interest period even though it is not legally in arrears relative to the terms of the contract.

the moves in monetary policy that are the primary driver of money-market rates, although an in advance structure will involve a one-period delay in the timing of when such moves are reflected.

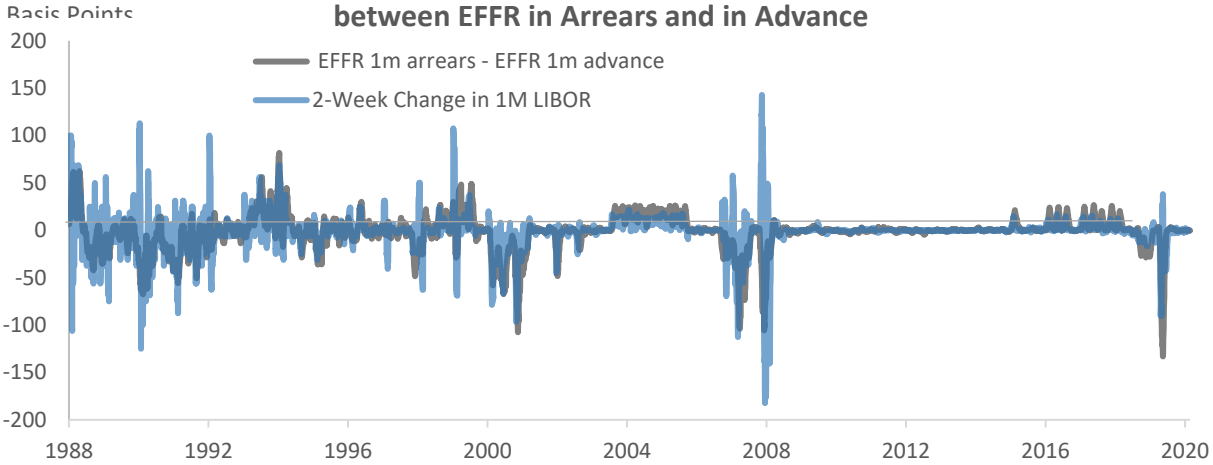
Figure 5: SOFR in Advance and in Arrears



Source: Federal Reserve Bank of New York; Federal Reserve Board staff calculations. Data from August 2014 to March 2018 represent modeled, pre-production estimates of SOFR.

Although lenders may view an *in advance* structure as less "up to date", this isn't an entirely new problem: LIBOR itself can often quickly become out of date, by about the same magnitude that an averaged overnight rate can. For example, in many loan contracts the borrower is able to draw on the loan at any time during an interest period based on the LIBOR rate that was set at the start of interest period. LIBOR rates are forward-looking, but even in a matter of weeks LIBOR can change radically and can itself become outdated. The amount of basis this creates is shown in Figure 6, and historically it has been quite large at times.[8] Figure 6 also shows the basis between a loan based on a compound average rate set in advance and one set in arrears. While it may seem counterintuitive, the historical magnitude of the basis that would have been caused by using a compound average overnight rate in advance rather than in arrears is comparable to the potential basis involved in LIBOR.

Figure 6: 2-Week Change in Monthly LIBOR versus the Difference between EFFR in Arrears and in Advance

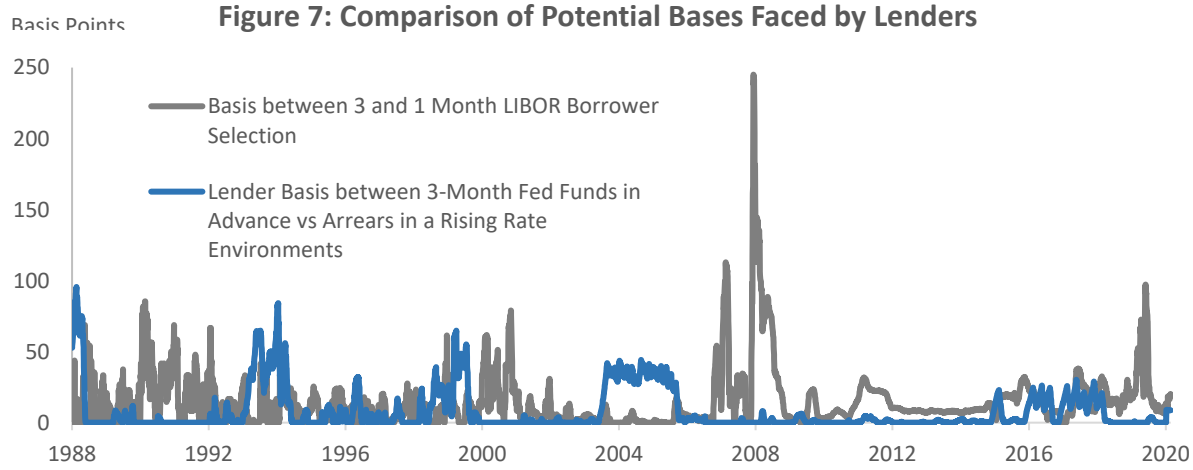


Source: Federal Reserve Bank of New York, ICE Benchmarks Administration; Federal Reserve Board staff calculations.

[8] Figure 6 shows the basis between the 1-month LIBOR rate set at the start of a monthly period and the 1-month LIBOR rate prevailing 15 days later.

LIBOR loans also have other sources of basis. Many LIBOR loan contracts allow the borrower to move between 1-, 3-, and 6-month LIBOR borrowing options at their discretion, which creates one-way risk of basis to the lender. Using an in advance rate, the lender will face a comparable sort of basis relative to in arrears if rates rise during the interest period. But as shown in Figure 7, that basis has typically been smaller than the basis that lenders routinely take on in LIBOR loans.

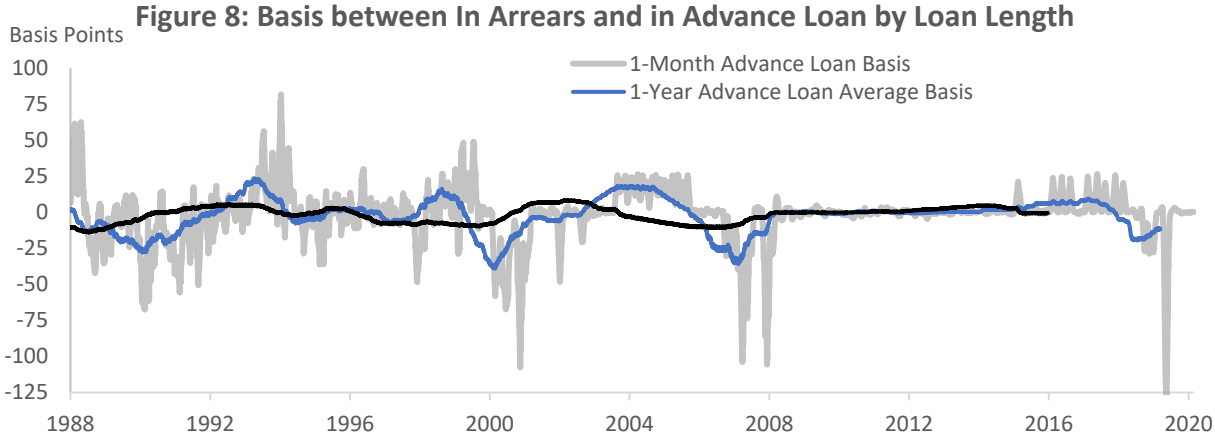
Figure 7: Comparison of Potential Bases Faced by Lenders



Source: Federal Reserve Bank of New York, ICE Benchmarks Administration; Federal Reserve Board staff calculations.

The amount of basis between *in arrears* and *in advance* conventions will depend on whether interest rates happen to be trending up or down over a given period. These differences will also depend on how frequently payments are made: the difference between an average of rates over the past month and an average of rates over the next month will typically be small, but the difference between an average of rates over this year and an average of rates over the next year may be larger because rates can move by more over a year than they typically might over a month. However, although in any given period there may be differences and investors may either gain or lose from one structure relative to the other, any movement in rates that are not reflected in the current interest period using an *in advance* rate will be paid in the following interest period (see Figure 4). On average, any differences will therefore tend to net out over the life of a loan or financial instrument if it lasts more than a few years. As shown in Figure 8, even on a loan that lasts only a year, the basis relative to in arrears is considerably smaller than the spot 1-month basis, and on a four-year loan the basis is minimal and comparable to the difference between compound and simple rates.

Figure 8: Basis between In Arrears and in Advance Loan by Loan Length



Source: Federal Reserve Bank of New York, ICE Benchmarks Administration; Federal Reserve Board staff calculations.

Any potential differences in return to the lender can be further minimized, however, through a "hybrid" structure that combines an in advance payment setting with adjustments, which is added either to the subsequent payment or to outstanding principal, in order to provide an overall return to the lender that is close to in arrears.

In these hybrid alternatives, the timing and frequency of payments would match the current structure of LIBOR loans and the timing and frequency of payments in a basic SOFR in advance loan. In order to make them easier to implement, there would be no adjustment at the time of the final payment in these structures.[9] These hybrid structures are thus a form of an *in advance* loan, but with an adjusted effective rate of interest.

There are two potential ways to adjust for the *in advance* rate set at the start of an interest period:

- *Interest Rollover*: Payments are set *in advance* and any missed interest relative to *in arrears* is rolled over into the payment for the next period.

  In this model, the payment for the period is set using an average of SOFR calculated at the start of the interest period; however the amount of interest due is calculated based on the average of SOFR over the interest period (*in arrears*), and any difference between the amount of interest paid and the interest accrued is simply rolled over into the payment for the next interest period. In this model, the remaining principal on the loan would not change.

- *Principal Adjustment*: Payments are set *in advance*, but principal and interest accrue *in arrears*.

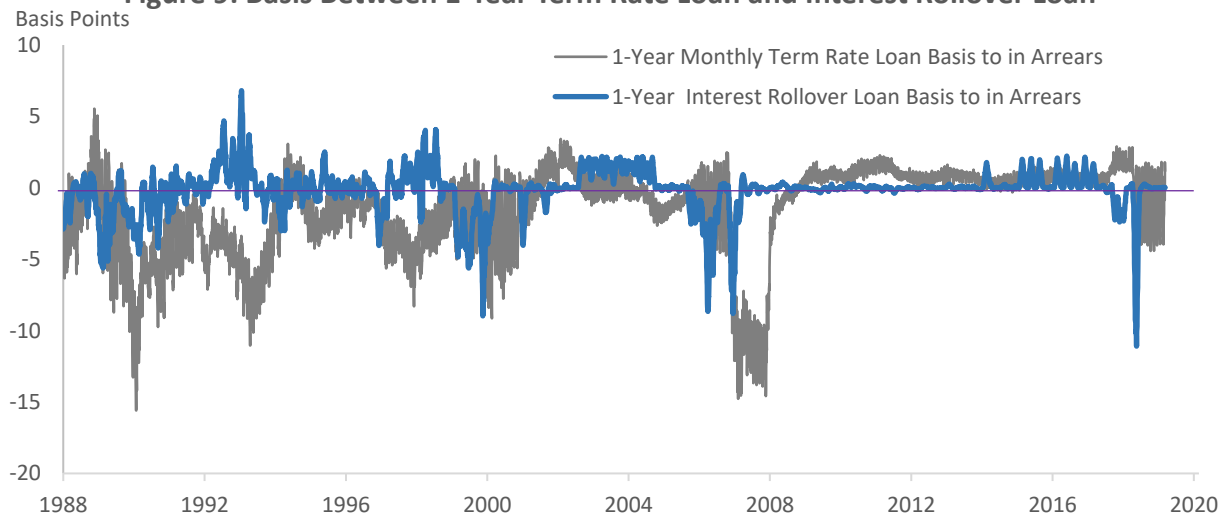
  In this model, the payment for the period is set using an average of SOFR calculated at the start of the interest period (*in advance*); however the amount of that set payment that is applied to interest will be based on the average of SOFR over the interest period (*in arrears*). In this model, the remaining principal on the loan would change over time based on the difference between the *in advance* and *in arrears* calculations for each period – if rates moved up over the interest period, then more of the payment would go to cover interest expenses and remaining principal would be higher, while if interest rates moved down then remaining principal would be lower.

The hybrid models will be unfamiliar to some, although it is not unusual to roll over certain payments into the next period as suggested with the *Interest Rollover* model, and amortizing loans adjust the amount of outstanding principal each period as suggested with the *Principal Adjustment* model. Both of these hybrids are designed to give borrowers ample advance notice of the payments they will need to make, while also structuring principal and interest to match the kind of *in arrears* return that investors may prefer. As shown in Figure 9, a hybrid model can potentially minimize the basis faced by investors while at the same time structuring payments in a way that borrowers could feel comfortable with. As shown, a hybrid approach can have a smaller basis return relative to in arrears than even a forward-looking term rate might because a forward-looking term rate incorporates expected changes in

[9] This introduces some basis relative to in arrears, but as shown in Figure 9, the basis is small, and it is operationally easier not to adjust for the last period. As can be seen from Figure 4, the difference in the return between an in advance and in arrears loan will be based on the difference between the average SOFR rate at the start of the loan (which is what the first payment of an in advance loan will be based on but would not be part of the payment of an in arrears loan) and the average at the end of the loan (which is what the last payment that an in arrears loan will be based on but would not be part of the payment in an in advance loan). With a hybrid structure, because the difference between in arrears and in advance is made up through the adjustment for every period except the very last, the difference in return is essentially limited to the difference between the average of SOFR at the start and end of the last payment period of the loan.

monetary policy, but will miss any unexpected changes, while hybrid loans will make up for any unexpected changes through the adjustments.

Figure 9: Basis Between 1-Year Term Rate Loan and Interest Rollover Loan



Source: Federal Reserve Bank of New York, Federal Reserve Board staff calculations.

Table 2 demonstrates how an Interest Rollover loan would have worked over 2007-08, when interest rates declined sharply, and Table 3 does the same for the Principal Adjustment approach. Although there would have been a 26 basis point difference between an *in advance* and an *in arrears* loan, the return differential of the hybrid loans would have only been 2-3 basis points relative to *in arrears*.

Table 2: Example of Interest Rollover approach on a 1-year loan over the steep drop in rates in 2007-08

				Interest Rollover Hybrid
Interest Determination Date	SOFR in Arrears	SOFR in Advance	Rollover Adjustment (bp)	Monthly Rate (SOFR in Advance + True Up)
4/26/2007	5.12%	5.21%	-9	5.21%
5/28/2007	5.10%	5.12%	-2	5.03%
6/27/2007	5.04%	5.10%	-6	5.08%
7/27/2007	4.67%	5.04%	-37	4.98%
8/28/2007	4.94%	4.67%	27	4.30%
9/27/2007	4.60%	4.94%	-34	5.21%
10/29/2007	4.25%	4.60%	-35	4.26%
11/28/2007	3.90%	4.25%	-35	3.90%
12/28/2007	3.45%	3.90%	-45	3.55%
1/29/2008	2.59%	3.45%	-86	3.00%
2/28/2008	1.78%	2.59%	-81	1.73%
3/31/2008	2.12%	1.78%	34	0.97%
Annualized Rate of Return	3.96%	4.22%		3.94%
Basis to in Arrears (bp)	---	-26		2

Table 3: Example of Hybrid Principal Adjustment approach on a 1-year loan over the steep drop in rates in 2007-08

				Principal Adjustment Hybrid	
Interest Determination Date	SOFR in Arrears	SOFR in Advance	Difference (bp)	Monthly Rate (SOFR in Advance)	Principal (Diff. Applied to Principal)
4/26/2007	5.12%	5.21%	-9	5.21%	$1,000,000.00
5/28/2007	5.10%	5.12%	-2	5.12%	$999,925.00
6/27/2007	5.04%	5.10%	-6	5.10%	$999,908.33
7/27/2007	4.67%	5.04%	-37	5.04%	$999,858.34
8/28/2007	4.94%	4.67%	27	4.67%	$999,550.05
9/27/2007	4.60%	4.94%	-34	4.94%	$999,774.95
10/29/2007	4.25%	4.60%	-35	4.60%	$999,491.68
11/28/2007	3.90%	4.25%	-35	4.25%	$999,200.16
12/28/2007	3.45%	3.90%	-45	3.90%	$998,908.73
1/29/2008	2.59%	3.45%	-86	3.45%	$998,534.14
2/28/2008	1.78%	2.59%	-81	2.59%	$997,818.52
3/31/2008	2.12%	1.78%	34	1.78%	$997,144.99
Annualized Rate of Return	3.96%	4.22%		3.93%	
Basis to in Arrears (bp)	---	-26		3	

D. In Arrears Conventions

Given the timing of when SOFR is published, the borrower would only have a few hours' notice before payment was due using a pure *in arrears* structure. Most borrowers would need more time than this, and there typically is some convention when using an *in arrears* structure that gives borrowers sufficient notice of the amount due before they are required to make a payment. There have been a number of conventions developed in order to allow for this, which we illustrate in Table 4 and describe in more detail below.

Table 4: Models for Using SOFR in Arrears

	Day 1 (First Day of Interest Period)	Day 2 SOFR for Day 1 Published	…	Day T-2 SOFR for Day T-3 Published	Day T-1 SOFR for Day T-2 Published	Day T (Last Day of Interest Period) SOFR for Day T-1 Published	Day T+1 (First Day of Next Period) SOFR for Date T Published	Day T+2
Plain Arrears	Use SOFR for Day 1	Use SOFR for Day 2	…	Use SOFR for Day T-2	Use SOFR for Day T-1	Use SOFR for Day T	Payment Due	
Arrears with Payment Delay	Use SOFR for Day 1	Use SOFR for Day 2	…	Use SOFR for Day T-2	Use SOFR for Day T-1	Use SOFR for Day T	OIS generally settle at T+2	Payment Due
Arrears with 1-Day Lockout	Use SOFR for Day 1	Use SOFR for Day 2	…	Use SOFR for Day T-2	Use SOFR for Day T-1	Use SOFR for Day T-1	Payment Due	
Arrears with 1-Day Lookback	Use SOFR for Day 0	Use SOFR for Day 1	…	Use SOFR for Day T-3	Use SOFR for Day T-2	Use SOFR for Day T-1	Payment Due	

- *Plain Arrears*: As shown in Table 4, under a pure in arrears structure, the SOFR rate for each given day in the interest period would be applied to calculate interest for that business day, and interest would be paid on the first day of the next interest period.

  Given the publication timing for SOFR and most other RFRs, this has the disadvantage of requiring payment on the same day that the final payment amount is known, and as a result it is often not operationally practical.

- *Payment Delay*: Interest is calculated in the same way as in a plain arrears framework, with the SOFR rate for each given day in the interest period applied to calculate interest for that business day, but interest is paid *k* days after the start of the next period.

  The payment delay structure matches and is easily hedged with standard SOFR OIS swaps, which generally use a payment delay to settle 2 days after the end of the interest period (often referred to as "T+2"). The advantage of this structure is that it gives more time for payment while still reflecting the movements in interest rates over the full interest period. The fact that payment is delayed would be reflected in the rate charged on the instrument, but nonetheless some investors may dislike any delay or find that the payment timing introduces mismatches with other payments.

- *Lockout or Suspension Period*: For most days during the interest period, interest is again calculated in the same way as in a plain arrears framework, with the SOFR rate for each given day applied to calculate interest for that business day; however, the SOFR rate applied for the last *k* days of the interest period is frozen at the rate observed *k* days before the period ends.

  A 2-5 day lockout has been used in some SOFR FRNs. A lockout allows the final interest amount due to be known *k* days in advance of the payment date, but for most of the interest period the daily interest rate applied will correspond to the most recent published value of the SOFR, which brings the calculation of net asset value and discounting closer to par value and may be important to some investors. A lockout does create some hedging basis relative to the market standard SOFR OIS structure, because it effectively skips the last *k* days of rates each interest period. However, dealers may be able to offer customized over-the-counter derivatives with lockouts to facilitate client hedging, and the investors who have tended to prefer a lockout structure may be less likely to hedge these investments. Because a lockout is designed to provide advance notice of payment only at the end of an interest period, it also may not be the best convention for a loan contract, where the loan could be repaid at any point in time and not only at the end of an interest period.

- *Lookback*: For each day in the interest period, the SOFR rate from *k* business days earlier is used to accrue interest.

  A 3-5 day lookback has been used in SONIA FRNs and is also used in many SOFR FRNs. Market participants may find a lookback helpful when there is a need to calculate interest accruing during an interest period, for example primary and secondary market trading or prepayments, and where more time is needed for such calculations.

  There are actually several forms of lookbacks, which we lay out below.

  - *Lookback without observation shift*

  If the lookback is for *k* days, then the observation date is *k* business days prior to the interest date. In a lookback without an observation shift, all other elements of the calculation are kept the same and the reference to a previous SOFR rate is the *only* change made.

Using an example of a 5-day lookback without observation shift in calculating interest for Tuesday, July 2, the SOFR rate for June 25 (5 business days prior to July 2) would be applied for 1 business day until Wednesday, July 3, while in calculating interest for Wednesday, July 3, the SOFR rate for June 26 (5 business days prior to July 3) would be applied for 2 business days until Friday, July 5.[10]

FRBNY SOFR DATA

DATE	Calendar Days Until Next Business Day	RATE (PERCENT)
Mon, Jun 24, 2019	1	2.39
Tue, Jun 25, 2019	1	2.41
Wed, Jun 26, 2019	1	2.43
Thu, Jun 27, 2019	1	2.42
Fri, Jun 28, 2019	3	2.5
Mon, Jul 1, 2019	1	2.42
Tue, Jul 2, 2019	1	2.51
Wed, Jul 3, 2019	2	2.56
Fri, Jul 5, 2019	3	2.59
Mon, Jul 8, 2019	1	2.48
Tue, Jul 9, 2019	1	2.45

Lookback without observation shift: The date that the SOFR rate is pulled from (the observation date) is *k* business days before the date that interest is applied (the interest date) and is applied for the number of calendar days until the next business day following the interest date.

Example of a 5-business day lookback: The rate for June 25 is applied on July 2 for one day, while the rate on June 26 is applied on July

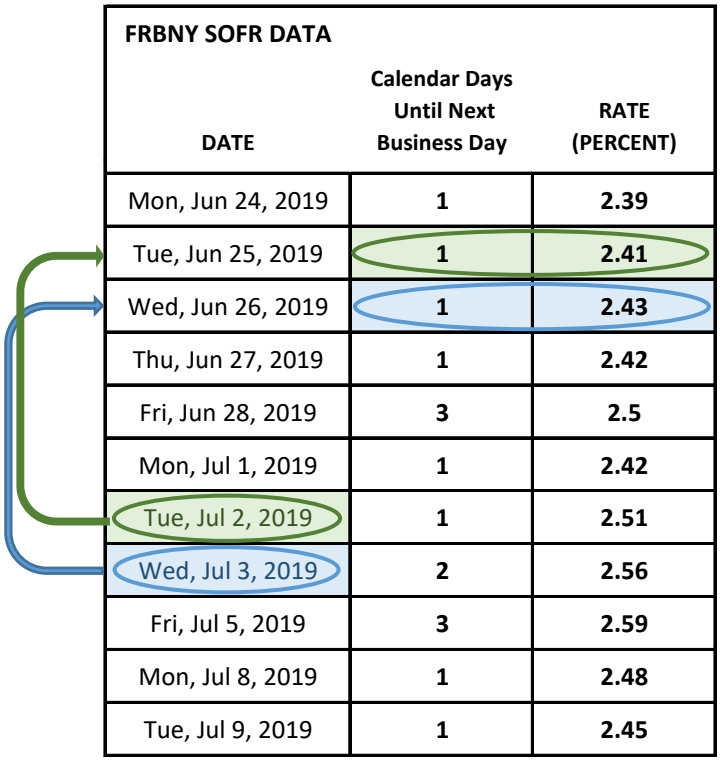
If the interest date is t, then a 5-day lookback will use the SOFR rate from the observation date t-5 (r_{t-5}) and it will apply that rate for the number of calendar days until the next business day following date t (n_t). The effective rate (i_t), which is the rate that is used in calculating daily accruals, is the SOFR rate on the observation date (r_{t-5}) multiplied by the number of days the rate applies for (n_t) and divided by the standard U.S. money market daycount convention of N =360.

- *Lookback with observation shift*

A lookback *with* observation shift also applies the SOFR rate from some fixed number of business days prior to the given interest date, but in contrast to a lookback without a shift, it applies that rate for the number of calendar days until next business date following the *observation date.*

[10] The ARRC has released a set of spreadsheets along with these technical Appendices in order to aid market participants as they test their implmentation of various conventions. An example of a 5-business day lookback is included in the file ARRC BWLG Example - Lookback without Observation Shift.xlsx, and a segment of the spreadsheet is shown below. As in the example above, in order to implement a lookback without observation shift, the only change in calculations in the spreadsheet relative to no lookback is that the observation date is 5-business days earlier than the interest date.

Continuing the example, using a 5-day lookback with observation shift in calculating interest for Tuesday, July 2, the SOFR rate for June 25 (5 business days prior to July 2) would be applied for 1 business day until Wednesday, July 3, while in calculating interest for Wednesday, July 3, the SOFR rate for June 26 (5 business days prior to July 3) would be applied for 1 business day.



FRBNY SOFR DATA		
DATE	Calendar Days Until Next Business Day	RATE (PERCENT)
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Mon, Jul 1, 2019	1	2.42
Tue, Jul 2, 2019	1	2.51
Wed, Jul 3, 2019	2	2.56
Fri, Jul 5, 2019	3	2.59
Mon, Jul 8, 2019	1	2.48
Tue, Jul 9, 2019	1	2.45

Lookback with observation shift: The date that the SOFR rate is pulled from (the observation date) is *k* business days before the date that interest is applied (the interest date) and is applied for the number of calendar days until the next business day following the observation date.

Example of a 5-business day lookback with observation shift: The rate for June 25 is applied on July 2 for one day, and the rate on June 26 is applied on July 3 for one day.

As discussed in the following box, the fallbacks in ISDA's IBOR protocol incorporate a lookback with observation shift, although a somewhat different variant than described here.

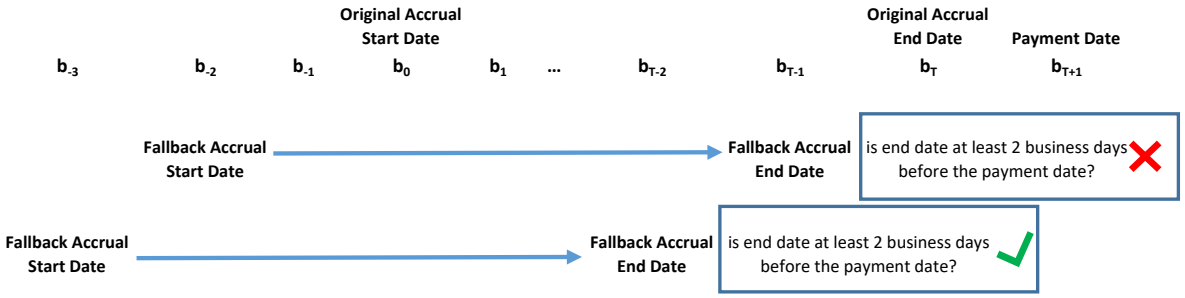
- *Interest-Period Weighted Observation Shift*

As just described, with an observation shift, interest is accrued according to the number of days in the observation period, which may differ from the number of days in the interest period. In some instances, parties in the FRN market choose to calculate interest payments using the annualized lookback rate with observation shift, but then to apply that annualized rate to the number of days in the interest period. A version of this interest-period weighted observation shift approach has also been noted as a potential convention by the Sterling Risk Free Rate Working Group for the sterling loan market, though it is not the principle recommendation and is discussed but not recommended in the ARRC's conventions for business loans. One issue with this approach, perhaps of particular importance to the loan market, is that it can at times result in a negative daily accrual even if SOFR rates are positive. This approach, and potential methods of accruing interest under it, are discussed further in Appendix 4.

ISDA's Lookback Structure

The fallback to compound SOFR in arrears that will be implemented through the ISDA protocol will have a lookback with observation shift, but it will differ in some respects from the lookback structures that is being used in SOFR cash products. In the structures laid out above, the start and end dates of interest accrual are fixed and for each business day in the period the SOFR rate from *k* days earlier is applied. In the ISDA lookback implementation, the fallback accrual *start date* is instead shifted back in order to allow the choice on an end date that is *at least k* business days before the LIBOR payment date, and the SOFR rate for each business day is applied. When the starting date is set, the end date then is chosen based on a modified following business day convention, for example, in a 3-month LIBOR swap, the end date will be 3-months modified following from the start date.

Most of the time, the new start date will be *k* days before the original LIBOR accrual starting date and the accrual calculations would equal those of the lookback with observation shift laid out in this Users Guide; however, there will be occasions in which the start date in the ISDA structure will be more than *k* days before, and also occasions when the end date will be more than *k* days before the payment date.



As an example, consider a 1-month LIBOR interest period that starts on January 29, 2021 and ends on February 26, 2021, with payment due that end date. A SOFR start date either 2 or 3 days earlier then the LIBOR start date, January 27 or January 26, would still have an end date of February 26, and a start date of Jan 25 would have a corresponding end date of Feb 25, which is only one day before the payment date. In order to provide at least 2 business days' notice, the start date would need to begin on January 22, with a corresponding end date of Feb 22, which is 4 business days before the payment date.

A lookback with observation shift is one of the conventions that has been recommended by the ARRC for FRNs.[11] However, the ARRC has recommended a lookback without shift for syndicated loans, which aligns with the approach recommended by the Sterling Risk Free Rates Working Group for Sterling markets. As discussed in the ARRC's conventions, syndicated loans have several complicating

[11] This convention is described under Two-Day Backward Shifted Observation Period and No Lockouts in the ARRC's SOFR Floating Rate Notes Conventions Matrix. See https://www.newyorkfed.org/medialibrary/Microsites/arrc/files/2019/ARRC_SOFR_FRN_Conventions_Matrix.pdf.

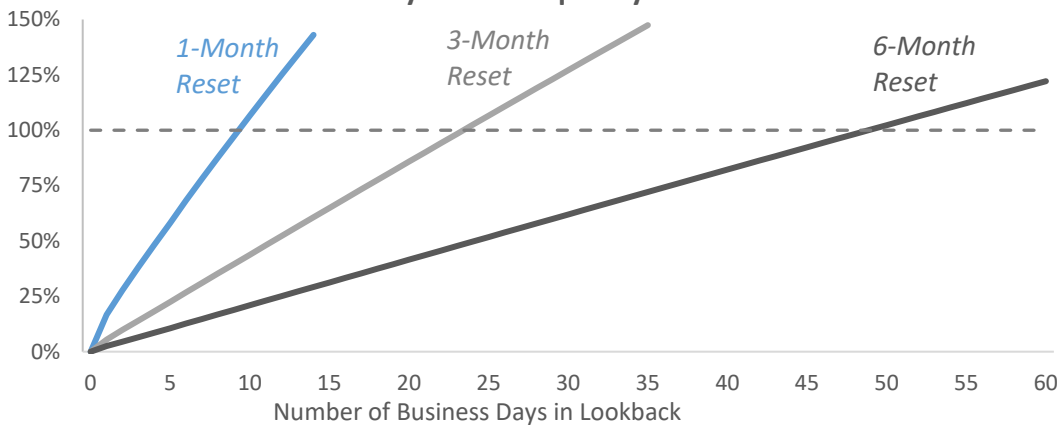
features that FRNs do not – principal can typically be repaid at any time, and syndicated loans are frequently traded between lenders and they do not trade clean.

The fact that principal may be repaid or that a lender may trade out of a loan before the end of an interest period makes implementing an observation shift more difficult in the loan market. For instance, in the example above, on July 3 interest is only charged for one day even though it would be two days until interest was paid. A lender who bought in to the loan on July 3 and sold out on July 5 may consider that they have been less than fully compensated given that they have provided some amount of principal for two days but only receive interest for one day. Or consider what was meant to be a monthly loan that began on July 8 but was repaid the next day. Under a 5-business day lookback with observation shift, the borrower would be charged for three day's interest based on the SOFR rate for Friday, June 28, even though they had only borrowed money for one day and should therefore only be charged for one day's interest.

Without trading or without early repayment, these discrepancies would average out and would be inconsequential. Because principal is constant in FRNs (and because they trade clean, meaning that the purchaser receives the full coupon), an observation shift is more easily implemented. With trading and the possibility of early repayment, these kinds of discrepancies may be more problematic, and the ARRC Business Loans Working Group members felt that a lookback with observation shift would not be the most appropriate convention for the syndicated loan market.[12]

Although each of these conventions have some benefit, in general lookback structures (with or without an observation shift) have been most widely used. FRNs have tended to have a shorter lookback period of 2-3 business days, while the ARRC's conventions for business loans contemplate a 5-business day lookback and securitizations using in arrears with a lookback might employ a longer lookback period.

Figure 10: Root Mean Spot Basis Relative of in Arrears Rates to Term Rates by Reset Frequency and Lookback



Source: Federal Reserve Bank of New York, Federal Reserve Board staff calculations.

[12] An analogy would be the difference between renting an apartment and staying at a hotel. Under a rental agreement, rent is the same each month even though some months have 28 days and others have 31 days, but the differences average out and people feel free to ignore them. In contrast, someone staying at a hotel is much more likely to take offense if they are charged for 3 days but only stayed 1 day or if they are charged a weekend rate when they stayed on a weekday.

Regardless, any of these lookback lengths will generally produce more accurate results than a forward looking term rate. Based on historical EFFR and EFFR OIS data, Figure 10 plots the root mean basis of different lookback lengths relative to the root mean basis of a forward-looking term rate. For a contract with a one-month reset frequency, a lookback of 9-10 days or less will be at least as accurate as any potential term rate. For a three-month reset frequency, a lookback as long as a month will be more or as accurate as a term rate, and for a six-month reset, the lookback frequency could be up to two months.

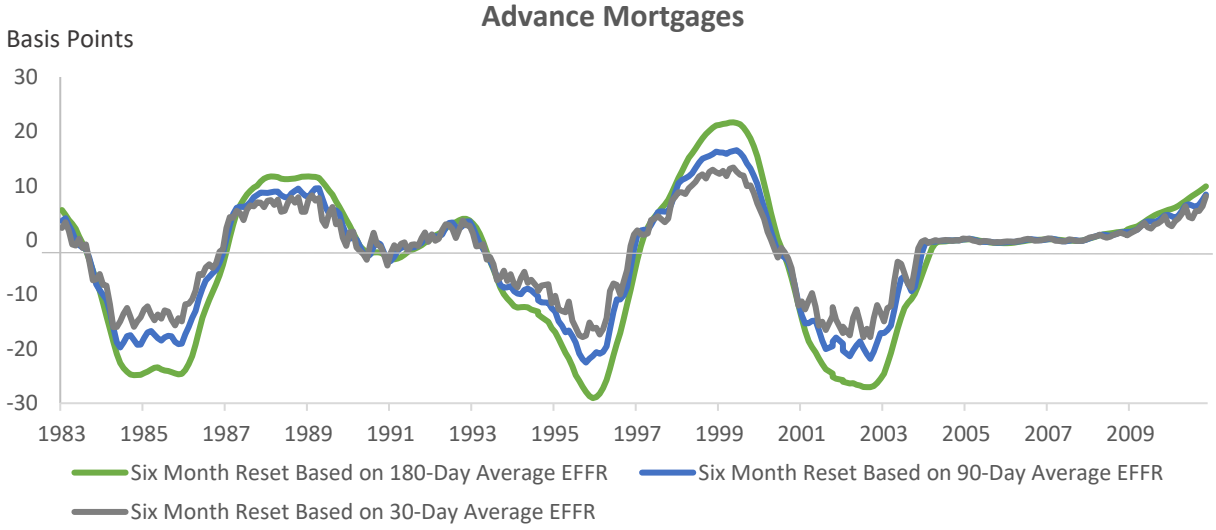
E. In Advance Conventions

Relative to in arrears, in advance structures are easier to implement, but there are still some choices involved implementing these structures. The two most familiar methods of implementing an in advance structure are the last reset and last recent methods:

- *Last Reset*: Use the averaged SOFR over the last interest reset period as rate for current interest period
- *Last Recent*: Use the averaged SOFR from a shorter recent period as rate for current interest period

Comparing these two *in advance* conventions, the last reset model is similar to a lookback model and will more closely match the structure of an OIS (although the payment structure will be lagged). For parties wishing to match payments (albeit, receiving the payment on the OIS contract prior to the due date for the loan payment) a last reset may be preferred. On the other hand, the last recent model is likely to have less basis relative to the *in arrears* average interest rate over the current interest period. This can be seen in Figure 11, which compares the basis between different models of Last Reset/Last Recent for different payment frequencies on a hybrid adjustable rate mortgage to a hypothetical *in arrears* structure.[13] The ARRC's Whitepaper on Using SOFR in Adjustable Rate Mortgages proposes a last recent structure, with a 6-month reset based on either 30- or 90-day Average SOFR. As can be seen in the figure, using a 30-day average with a 6-month reset comes close in terms of basis to an even shorter 3-month last reset structure.

Figure 11: Comparing Bases to In Arrears for Different Models of In Advance Mortgages



Source: Federal Reserve Bank of New York, Federal Reserve Board staff calculations.

[13] In these mortgage simulations, a hypothetical 5/1 Adjustable Rate Mortgage that refinances in year 8 of the mortgage is considered, with floating rate payments based on historical values of EFFR. As described earlier, in these and following simulations, the basis is calculated as the spread (expressed as an annual rate) that would need to be added to the *in advance* instrument in order to equate the ex post net present value of payments received with the *in arrears* instrument. Net present values are calculated using the internal rate of return on the *in arrears* instrument. A positive basis implies that investors would have required added compensation to have broken even on the *in advance* instrument, while a negative basis implies that investors would have gained from the *in advance* instrument and would have had to rebate some of the interest received to have broken even relative to *in arrears*.

F. The ARRC's Conventions Recommendations Documents

The ARRC has produced several sets of convention and term sheet documents for specific cash products, both in arrears (for FRNs, and syndicated and bilateral business loans) and in advance (for intercompany loans and consumer products). The table below provides links to these documents. In addition, Appendices 4-6 include draft term sheets for business loans based on simple interest in arrears, SOFR in advance, and an Interest Rollover SOFR loan.

Table 5: ARRC Conventions and Term Sheet Documentation

Product	Conventions/Term Sheet Document
Floating Rate Notes	Compound SOFR in Arrears with Lookback (No Observation Shift) Compound SOFR in Arrears with Lookback and Observation Shift Compound SOFR in Arrears with Payment Delay Compound SOFR with Index Calculation
Syndicated Business Loans	Compound SOFR in Arrears with Lookback Simple SOFR in Arrears with Lookback
Bilateral Business Loans	Compound SOFR in Arrears with Lookback Simple SOFR in Arrears with Lookback
Intercompany Loans	30- or 90-Day SOFR in Advance
Adjustable-Rate Mortgages	30- or 90-Day SOFR in Advance
Student Loans	30- or 90-Day SOFR in Advance

G. The Interaction between SOFR and the Forward-Looking Term Rate

As noted above, the FSB has been clear in its assessment that financial stability will be enhanced if use of forward-looking term rates is narrow and most market participants move toward use of RFRs, while also recognizing the potential usefulness of forward-looking RFR-based term rates as a fallback rate for legacy contracts and in cash markets in certain circumstances.

While the ARRC has set a goal of seeing a forward-looking term SOFR rate, production of a term rate can only be guaranteed if most new products use SOFR directly, as otherwise SOFR derivatives markets are unlikely to achieve or maintain the depth needed to produce a robust term rate. It is also important that market participants are also clear on what the forward-looking term SOFR rate is expected to be, and its relationship to the overnight SOFR, in order to understand where use of such rates could be best made.

While the overnight Treasury repo market underlying SOFR is extraordinarily deep, term repo markets are much thinner, and it would not be possible to build a robust, IOSCO-compliant rate directly off the term Treasury repo market. As discussed in the ARRC's Second Report, there is really no term cash market in the United States with enough depth to build a reliable, robust, transactions-based rate produced on a daily basis that would be able to meet the criteria that the ARRC set in choosing SOFR. Therefore, the ARRC has proposed that a private administrator could construct a forward-looking term rate based on SOFR derivatives markets once those markets develop enough liquidity. Because SOFR derivative markets have developed quickly and are expected to achieve a very high degree of liquidity, it is reasonable to expect that these markets will eventually be sufficiently liquid and robust to construct a forward-looking term rate, but the timing cannot be guaranteed.

Under the ARRC's proposal, the forward-looking term rate would be based on some combination of SOFR futures and SOFR OIS transactions.[14] The ARRC has not endorsed a specific methodology for producing these rates, but a recent working paper has laid out one potential methodology and the authors have released a series of "indicative" term rates that may help to promote better understanding as to how rates of this type might behave over time.[15] As liquidity in SOFR derivatives markets continues to develop, the ARRC anticipates that private vendors will seek to produce one or more forward-looking term rates for commercial use, which the ARRC has committed to evaluate with the aim of recommending one such rate provided that it satisfies the ARRC's criteria.

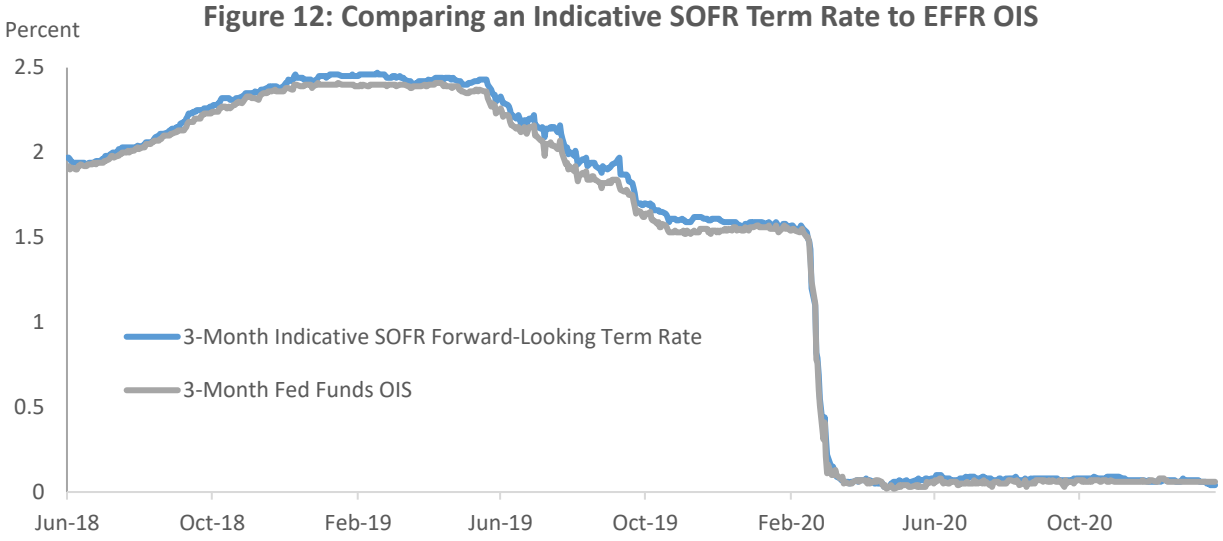
Any forward-looking term rates are expected to be equal or close to the underlying SOFR OIS curve. An OIS contract involves exchanging a set of fixed-rate payments for a set of floating-rate payments between two parties. The floating rate is a compound average of the overnight rate calculated over the interest period, while the fixed rate is set at the start of the period. If we call $OIS_{3m}(t)$ the fixed rate on a 3-month OIS contract entered into at date *t*, then the 3-month forward-looking term rate would be either equal to $OIS_{3m}(t)$ or close to it. The same would be true for the potential 1-month

[14] These two markets are very tightly linked together. SOFR futures pay an average of SOFR over a given month or quarter, for example, the average of SOFR realized over the month of June or the average over the first quarter of the year. SOFR OIS pay the compounded average of SOFR over a fixed period of time, for example, a one-month OIS contract beginning on March 15 would pay the compound average of SOFR realized over the period from March 15 to April 14.

[15] See Heitfield and Park (2019), Inferring Term Rates from SOFR Futures Prices, FEDS discussion paper 2019-014. Further description of the methodology as well as a data file that presents indicative forward-looking term rates derived from end-of-day SOFR futures prices and compound averages of daily SOFR rates can be found in Heitfield and Park (2019), Indicative Forward-Looking SOFR Term Rates, a staff FEDS Note published April 14, 2019. These rates are presented for informational purposes only and are not appropriate for use as reference rates in financial contracts.

or 6-month counterparts, $OIS_{1m}(t)$ and $OIS_{6m}(t)$. Figure 12 compares the indicative SOFR term rate to an OIS rate referencing EFFR, and one can see that they move quite closely together.

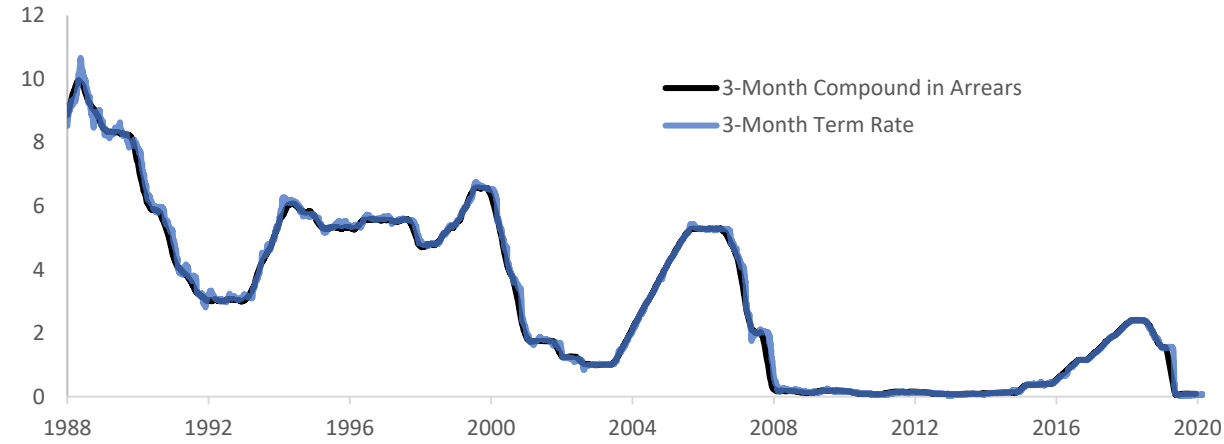
Figure 12: Comparing an Indicative SOFR Term Rate to EFFR OIS



Source: Refinitiv, Federal Reserve Board staff calculations.

In general, there will be a tight economic link between the forward-looking term rate and the compound average of SOFR in arrears used as the floating rate in OIS contracts. The fixed rate is set so that the OIS contract has zero value at the time it is entered into; that is, the value of receiving the fixed rate is exactly equal to the value of receiving the floating rate. In this sense, the fixed rate (the forward-looking term rate) will be economically equivalent to the corresponding expected compound average of SOFR. We don't have a long history of SOFR OIS yet, but Figure 13 shows this type of link between EFFR OIS and compound averages of the EFFR, as a proxy.

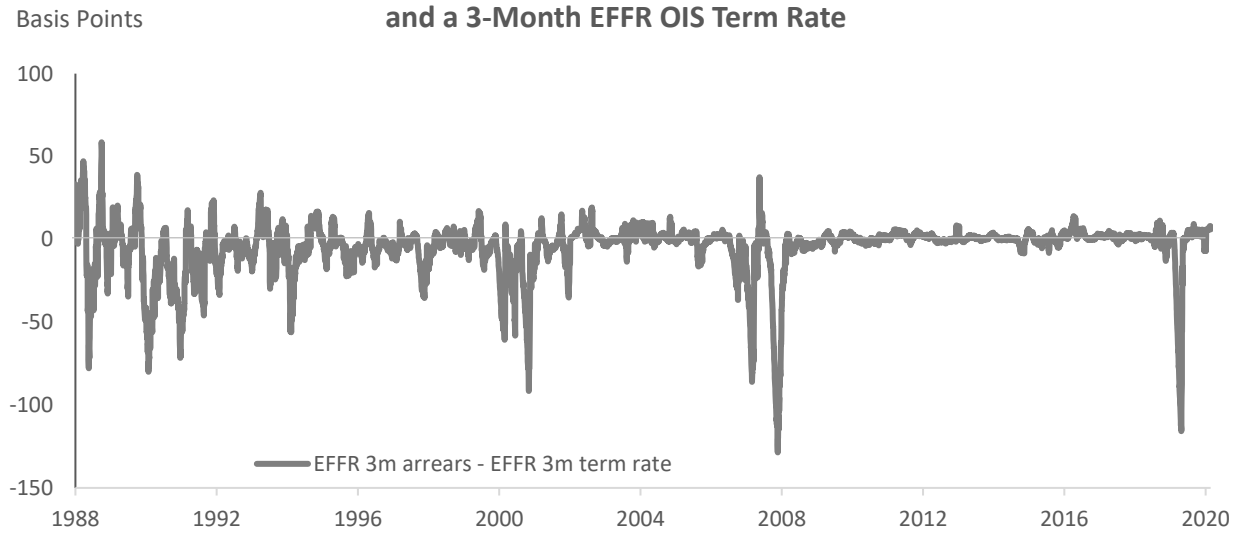
Figure 13: Comparing EFFR Term Rate and Compound Arrears



Source: Federal Reserve Bank of New York; Refinitiv; Federal Reserve Board staff calculations

The key difference between the two rates is that the term rate reflects market *expectations* as to what will happen to interest rates, while the compound average used in OIS contracts will reflect what *actually happens* to interest rates over the period. Although market expectations have generally been close to the actual movement in rates, they have not always matched. As shown below, the basis between the term rate and compound overnight rates has been material at times, in particular during times when rates have fallen rapidly.

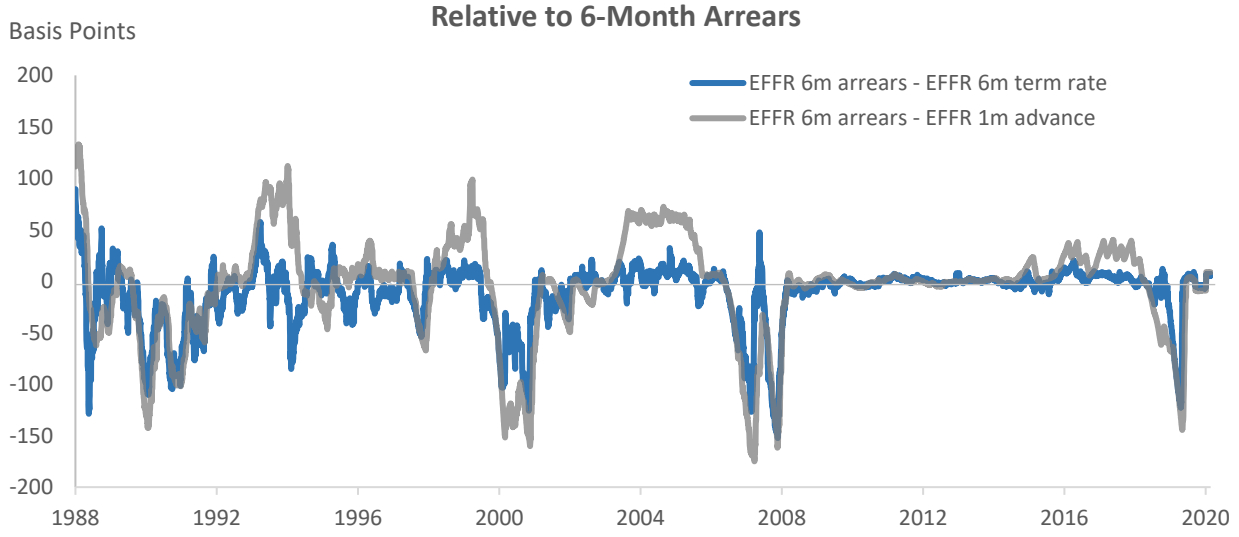
Figure 14: Basis between 3-Month Compound EFFR in Arrears and a 3-Month EFFR OIS Term Rate



Source: Federal Reserve Bank of New York; Refinitiv; Federal Reserve Board staff calculations

The amount of this basis will depend on the tenor of the term rate. As discussed above, the basis between an in advance and in arrears rate will increase with the length of the interest period. As rates are less predictable over longer periods of time, a longer-tenor term rate will be less able to match the actual movement in rates over the period. As shown below, a 6-month EFFR OIS rate has historically had close to the same basis to in arrears as a 30-day average of the EFFR set in advance.

Figure 15: Bases between a 6-Month Term Rate and an in Advance Rate Relative to 6-Month Arrears



Source: Federal Reserve Bank of New York; Refinitiv; Federal Reserve Board staff calculations

The basis involved imply that a forward-looking term rate would not generally be effectively hedged by a standard OIS contract. As discussed above, for in arrears and also in advance structures, which can be viewed as in arrears with a payment delay, hedging would be easier to achieve.

Of course, potential users of a forward-looking term rate may not wish to hedge their exposures, especially if they understand that hedging is easier with structures based directly on SOFR, but for those who might seek to hedge a term-rate exposure, it may be helpful to understand how forward-looking-term rate exposures can (and in an economic sense, will) be dynamically hedged by using SOFR OIS. To see this, consider an example of an end user that wants to hedge a set of quarterly term SOFR payments they are required to make over the next year by converting their floating term rate payments into fixed rate. Although they are paying the quarterly term rate, they could still hedge this directly in the SOFR OIS market with the following steps:[16]

- Step 1: Enter into a 12-month SOFR OIS contract at the start of the year to pay the fixed-leg rate $OIS_{12m}(t)$ and receive quarterly compound SOFR payments.

- Step 2: At the start of each quarter, enter into a 3-month SOFR OIS contract to receive the fixed-leg term rate, OIS_{3m} and pay compound SOFR over that quarter. Use the quarterly floating-rate SOFR payment from the 12-month OIS in Step 1 to pay the floating-rate leg of Step 2 and use the fixed rate payment of this swap to pay the quarterly term-rate owed.

In practice, many firms would engage a bank or a dealer to do these steps for them rather than taking it on themselves, and there would be some transaction cost to doing this. If they entered into a bespoke term rate swap with their dealer instead, then the dealer would need to enter the SOFR OIS market to hedge the swap and that would involve the same basic steps and costs. In addition, there may be some charges for any basis risk (the term rate benchmark may not precisely match the OIS rate that the dealer may be able to obtain on any given day), and there may be associated costs if the bespoke swap cannot be cleared or if the dealer needs to warehouse the swap and must charge for the associated risk. There may also be additional costs associated with hedging based on a SOFR term rate derivative given that, in contrast to SOFR itself, any potential SOFR term-rate benchmark is not included in the Financial Accounting Standards Board's (FASB's) hedge accounting list. Each of these factors would result in additional transactions costs to the parties to the transaction.

Dealers are equipped to provide these kinds of services to their clients, and presumably they will, but they will also need to pass on the associated costs. On the other hand, many of these costs could be avoided from the start if the borrower used SOFR rather than a forward-looking term-rate. An instrument that required payments of compounded SOFR could be directly hedged in the SOFR OIS market, with far fewer steps and costs. Which leads to the final important point of this Guide – *use of the forward-looking term rate will tend to involve more transactions costs than using SOFR, and if end users know that they want to hedge their floating rate payments then it would involve fewer transaction costs if they can modify their systems to be able to pay or receive the compound average SOFR rather than paying or receiving the forward-looking term rate.*

None of this is meant to contradict the idea that the forward-looking term rate can be a useful tool for some market participants, but it is also important that they understand the likely costs as well. A

[16] This example uses an OIS convention in which floating and fixed rate payments are paid quarterly.

number of firms will likely wish to avoid these costs and use SOFR from the start. Many other firms will likely come to the same conclusion over time as they gain experience with the new market structure and are able to update their systems to accommodate using SOFR.

Appendix 1: Calculating Compound Interest

For some, it may be useful to note the mathematical formulas behind compound and simple interest conventions. The table below demonstrates the basic distinction between the two concepts: with simple interest, interest is charged based only on the principal outstanding, while with compound interest, interest is charged based both on outstanding principal and accumulated unpaid interest.

Table A1: Calculating Simple and Compound Interest

Simple Interest on a One-Week SOFR Loan of $1 Million Drawn on Jan 7, 2019

	Secured Overnight Financing Rate (Percent, Annualized)	Number of Days Rate is Applied	Effective Rate (Not Annualized)	Principle	Principal + Accumulated Interest	**Interest Charge for Next Business Day (Effective Rate*Principal)**
Monday, Jan 7, 2019	2.41	1	0.0241/360 = 0.006694%	**$1,000,000.00**	$1,000,000.00	**$66.94**
Tuesday, Jan 8, 2019	2.42	1	0.0242/360 = 0.006722%	**$1,000,000.00**	$1,000,066.94	**$67.22**
Wednesday, Jan 9, 2019	2.45	1	0.0245/360 = 0.006806%	**$1,000,000.00**	$1,000,134.16	**$68.06**
Thursday, Jan 10, 2019	2.43	1	0.0243/360 = 0.006750%	**$1,000,000.00**	$1,000,202.22	**$67.50**
Friday, Jan 11, 2019	2.41	3	3*0.0241/360 = 0.020083%	**$1,000,000.00**	$1,000,269.72	**$200.83**
Monday, Jan 14, 2019	---	---	---	**$1,000,000.00**	$1,000,470.55	

Payment Due
Monday, Jan 14, 2019 **$1,000,470.56**

Annualized Simple Rate of Interest: $\left(\frac{360}{7}\right)\left[\frac{0.0241}{360}+\frac{0.0242}{360}+\frac{0.0245}{360}+\frac{0.0243}{360}+\frac{3\times 0.0241}{360}\right]$

= (360/7)*(.047056%) = 2.4200%

Compound Interest on a One-Week SOFR Loan of $1 Million Drawn on Jan 7, 2019

	Secured Overnight Financing Rate (Percent, Annualized)	Number of Days Rate is Applied	Effective Rate (Not Annualized)	Principle	**Principal + Accumulated Interest**	**Interest Charge for Next Business Day (Effective Rate*(Principal+Accumulated Interest))**
Monday, Jan 7, 2019	2.41	1	0.0241/360 = 0.006694%	$1,000,000.00	**$1,000,000.00**	**$66.94**
Tuesday, Jan 8, 2019	2.42	1	0.0242/360 = 0.006722%	$1,000,000.00	**$1,000,066.94**	**$67.23**
Wednesday, Jan 9, 2019	2.45	1	0.0245/360 = 0.006806%	$1,000,000.00	**$1,000,134.17**	**$68.06**
Thursday, Jan 10, 2019	2.43	1	0.0243/360 = 0.006750%	$1,000,000.00	**$1,000,202.23**	**$67.51**
Friday, Jan 11, 2019	2.41	3	3*0.0241/360 = 0.020083%	$1,000,000.00	**$1,000,269.74**	**$200.89**
Monday, Jan 14, 2019	---	---	---	$1,000,000.00	**$1,000,470.63**	

Payment Due
Monday, Jan 14, 2019 **$1,000,470.63**

Annualized Compound Rate of Interest: $\left(\frac{360}{7}\right)\left[\left(1+\frac{0.0241}{360}\right)\left(1+\frac{0.0242}{360}\right)\left(1+\frac{0.0245}{360}\right)\left(1+\frac{0.0243}{360}\right)\left(1+\frac{3\times 0.0241}{360}\right)-1\right]$

= (360/7)*(.047064%) = 2.4204%

ISDA's Compound SOFR formula is based on the following annualized rate calculation:

(1) $Compound\ Annualized\ Interest = \left[\prod_{b=1}^{T}\left(1+\frac{r_b \times n_b}{N}\right)-1\right]\frac{N}{d_c}$

Where

T = the number of *business days* in the interest period

d_c = the number of *calendar days* in the interest period[17]

r_b = the interest rate applicable on business day *b*

n_b = the number of calendar days for which rate r_b applies (on most days, n_b will be 1, but on a Friday it will generally be 3, and it will also be larger than 1 on the business day before a holiday). This can also be stated as the number of calendar days from and including business day *b* to but excluding the following business day.

N = the market convention for quoting the number of days in the year (in the United States, the convention for money markets is N = 360, while in the UK it is N=365).

And *b* represents a series of ordinal numbers representing each business day in the period.

Because it has caused some confusion, we lay out the conditions under which this type of "compound rate" equation can be used, and how to calculate compound interest more generally. To do so, we define a few additional terms for a given business date *t*:

i_t = the effective interest rate for date *t*

P_t = outstanding principal for date *t*

A_t = the accumulated unpaid accrued interest for date *t before* any interest paydown

A'_t = the accumulated unpaid accrued interest for date *t after* any interest paydown

PD_t = the amount of any interest paydown (a negative number, so that $A'_t = A_t + PD_t$)

The equations below would work with either an effective interest rate based on a lookback without observation shift ($i_t = \frac{r_t \cdot n_{t-k}}{N}$) or with no lookback ($i_t = \frac{r_t \cdot n_t}{N}$), as would be the case with the payment delay used in derivatives.

General Case: Compound Balance

Whereas under simple interest daily accrued interest depends only on the outstanding principal for that day:

$$A_{t+1} = A'_t + i_t \cdot P_t$$

[17] As discussed in more detail in section 2, the formula as written assumes that $d_c = \sum n_b$, which *will* be the case with the standard uses of the ISDA compound SOFR formula in derivatives, where notional principal is typically constant over an interest period and payment is made with a delay.

with compound interest, daily interest accrual is charged both on outstanding principal and on accumulated unpaid interest:

$$(2) \qquad A_{t+1} = A'_t + i_t \cdot [P_t + A'_t]$$

This formula is the basic definition of compound interest – interest is charged both on outstanding principal and accumulated unpaid accrued interest. Within the ARRC Business Loans Working Group, this equation (2) above has been termed the "Compound Balance" approach (i.e., compounding interest on the "balance due" on the instrument) to calculating compound interest. The Compound Balance approach can be applied regardless of whether principal changes or whether some portion of interest is repaid during an interest period.

The daily interest accrual under the Compound Balance approach is simply calculated by applying the appropriate day's SOFR rate to outstanding principal and accrued unpaid interest:

$$A_{t+1} - A'_t = i_t \cdot [P_t + A'_t]$$

The ARRC has published a spreadsheet ARRC BWLG Compounding Methods Examples.xlsx containing examples of the different methods of calculating compound interest. A screenshot of the worksheet with an example of compound balance is shown below. Implementation requires keeping track of accumulated interest as shown in the screenshot:

	A	F	G	H	I	J	K
1	SOFR Rate and Principal/Paydown Information				Compound Interest Accrual Calculations		
3					Accumulated Unpaid Interest *Before* Paydown (A_t)	Accumulated Unpaid Interest *After* Paydown (A'_t)	Daily Base Interest Accrual
4	Interest Date (t)	Principal (P_t)	Interest Paydown (PD_t)	Effective Rate $i_t = \frac{r_t \cdot n_t}{N}$	$A_{t+1} = A'_t + i_t \cdot [P_t + A'_t]$	$A'_t = A_t + PD_t$	$i_t \cdot [P_t + A'_t]$
12	July 9, 2019	$100,000,000.00		0.00681%	$56,400.74	$56,400.74	$6,892.30
13	July 10, 2019	$100,000,000.00		0.00683%	$63,210.14	$63,210.14	$6,809.39
14	July 11, 2019	$100,000,000.00		0.00669%	$70,047.79	$70,047.79	$6,837.65
15	July 12, 2019	$100,000,000.00		0.01967%	$76,746.92	$76,746.92	$6,699.13
16	July 15, 2019	$90,000,000.00	($9,642.87)	0.00683%	$96,428.68	$86,785.81	$19,681.76
17	July 16, 2019	$90,000,000.00		0.00686%	$92,941.74	$92,941.74	$6,155.93
18	July 17, 2019	$90,000,000.00		0.00686%	$99,123.12	$99,123.12	$6,181.38

Accumulated Unpaid Interest Before Paydown

Cell I18 =

J17+ H17*(F17+J17)

Accumulated Unpaid Interest After Paydown

Cell J18 =

I18 + G18

Daily Accrual

Cell K18 =

H17*(F17+J17)

Special Case: Compound Rate

While the Compound Balance approach can be applied generally, as discussed further in Box A.1, the "Compound Rate" approach should only be employed under specific conditions:

a) Principal remains constant within an interest period, or

b) If some portion of principal is repaid, then a corresponding proportion or accrued interest is repaid at the same time.

Under the specific conditions, the general formula can be simplified to the (non-annualized) version of ISDA's formula for Compound SOFR

$$(3) \qquad A_{t+1} = \mathrm{UCR}_t \cdot P_t$$

Where the term $\mathrm{UCR}_t = \left[\prod_{b=1}^{t}(1 + i_b) - 1\right]$ is called the Unannualized Cumulative Compound Rate.

Daily accrual can be calculated directly using this equation and equation for A'_t

$$A_{t+1} - A'_t = \mathrm{A}_{t+1} - \mathrm{A}_t - PD_t$$

but market participants have tended to prefer a variant of this calculation, the "Noncumulative Compound Rate" approach, which recognizes that the required relationship between that amount of interest paid down and any reduction in principal implies that this calculation for daily accrued interest can be simplified to:

$$A_{t+1} - A'_t = (UCR_t - UCR_{t-1}) \cdot P_t$$

The Compound Rate and the Noncumulative Compound Rate equations are special cases of the Compound Balance approach. While the Compound Balance approach will correctly accrue interest under general conditions, if the special conditions are not met, that is if principal is repaid but

interest is not, then the Compound Rate and Noncumulative Compound Rate approaches will not calculate accrued interest correctly.[18]

[18] There are, however, two possible workarounds to fix this problem:

(i) treat any unpaid interest as a new loan starting at date *t*, while treating the original loan as if interest had been repaid on it and using the Compound Rate approach with it.

(ii) carry a separate set of internal calculations accruing interest on the amount of principal that would be outstanding if the borrower's payment was proportionately allocated between principal and interest reduction.

In either case, the workarounds effectively reproduce the Compound Balance calculations and simply using the Compound Balance approach directly may arguably be more straightforward.

Appendix 2: The SOFR Index and Interpolating Interest on Non-Business Days

The SOFR Index is based on ISDA's Compound SOFR definition (see Appendix 1). According to that definition interest is compounded on each government securities trading business day based on that day's SOFR rate. On any day that is not a business day, simple interest applies, at a rate of interest equal to the SOFR value for the preceding business day. In accordance with broader U.S. dollar money market convention, interest is calculated using the actual number of calendar days, but assuming a 360-day year.

The SOFR Index measures the cumulative impact of compounding SOFR using the ISDA Compound SOFR definition on a unit of investment over time, with the initial value set to 1 on April 2, 2018, the first value date of the SOFR. The Index is cumulatively compounded by the value of each SOFR thereafter:

$$I_0 = 1$$

$$I_1 = \left(1 + \frac{r_0 \times n_0}{N}\right)$$

$$I_2 = \left(1 + \frac{r_0 \times n_0}{N}\right)\left(1 + \frac{r_1 \times n_1}{N}\right)$$

$$I_3 = \left(1 + \frac{r_0 \times n_0}{N}\right)\left(1 + \frac{r_1 \times n_1}{N}\right)\left(1 + \frac{r_2 \times n_2}{N}\right)$$

$$\vdots$$

$$I_t = \left(1 + \frac{r_0 \times n_0}{N}\right)\left(1 + \frac{r_1 \times n_1}{N}\right)\left(1 + \frac{r_2 \times n_2}{N}\right)\cdots \left(1 + \frac{r_{t-2} \times n_{t-2}}{N}\right)\left(1 + \frac{r_{t-1} \times n_{t-1}}{N}\right)$$

The Index for any date *t* can be written more compactly as:

$$I_t = \prod_{b=0}^{t-1}\left(1 + \frac{r_b \times n_b}{N}\right)$$

Or, recursively as:

$$I_t = I_{t-1} \times \left(1 + \frac{r_{t-1} \times n_{t-1}}{N}\right)$$

The value of the level of the Index on any given date is not generally of much direct use, but taking the ratio of two values of the Index can be used to calculate compounded averages of the SOFR over custom time periods between any two dates within the SOFR publication calendar. To calculate compounded interest over a period starting on a date *x* and ending on a date *y* (*y* being the date that interest would be due), the ratio of the Index between *x* and *y* can be used to calculate the amount:

$$\frac{I_y}{I_x} = \prod_{b=x}^{y-1}\left(1 + \frac{r_b \times n_b}{N}\right)$$

Noting this, the compound annualized rate of interest can be calculated as

$$Compound\ Annualized\ Interest\ between\ x\ and\ y = \left[\frac{I_y}{I_x} - 1\right]\frac{360}{d_c}$$

Where d_c is the number of calendar days in the calculation period

Given that the SOFR Index reflects the same arithmetic as the SOFR Averages, rates calculated using the SOFR Index with the same start and end dates as the SOFR Averages will effectively produce equivalent results. However, because the SOFR Index is rounded, averages calculated from Index values will not maintain the same precision as the SOFR Averages; as a result, minor differences from the published averages may occasionally occur at the fifth decimal place.

Because the SOFR Index implements the ISDA compound SOFR definition, it can be used to calculate accrued interest in a plain arrears convention and also in a convention with compound interest and lookback with observation shift. However, the Index cannot be used with other conventions, such as a lookback without shift or a lockout, nor can it be used to calculate simple interest accrual.

Calculating Accrued Interest over Non-Business Days

The ISDA compound SOFR definition and the SOFR Index are designed to calculate interest accruals on government-securities trading business days, the days when SOFR itself is published. However, they can be adapted to calculate interest accrual on a non-business day. The ISDA definition compounds interest on business days, but over weekends and holidays interest is quoted and treated as simple. For example, if the SOFR rate on a Friday is r, then the interest accrued through Monday is

$$I_{Monday} = I_{Friday} \cdot \left(1 + \frac{3r}{N}\right)$$

because there are 3 days between Friday and Monday (presuming that Monday is a business day). By the same logic, if interest was only accrued through Sunday rather than Monday, then the accrual would be

$$I_{Sunday} = I_{Friday} \cdot \left(1 + \frac{2r}{N}\right)$$

Which can also be written as $I_{Sunday} = \frac{1}{3} \cdot I_{Friday} + \frac{2}{3} \cdot I_{Monday}$

And if interest was only accrued through Saturday, then the rate would be

$$I_{Saturday} = I_{Friday} \cdot \left(1 + \frac{r}{N}\right)$$

Which similarly can also be written as $I_{Saturday} = \frac{2}{3} \cdot I_{Friday} + \frac{1}{3} \cdot I_{Monday}$

There are other conventions that could be used to accrue an implied amount of interest on a non-business day, but this convention seems most consistent with the logic of the compound SOFR definition.

Using this convention, the SOFR Index can be used to calculate compounded averages of the SOFR starting or ending on non-business days by interpolating between the published Index values that cover the relevant non-business day. Use of the SOFR Index would conform to a plain arrears framework, or to a lookback with observation shift, but the Index can't be used with a lookback without observation shift or a lockout; however, similar methods can be used to calculate interest on a non-business day for these conventions.

For interest periods *ending* on a non-business day, simply use a linear interpolation between the SOFR Index values immediately before and after the desired day, where the weights on the two indexes are the fraction of the break period *after and before* the desired date. For example, for a typical weekend, weight the SOFR Index values on the preceding Friday and following Monday as follows:

$$\frac{I_{Saturday}}{I_{start\ date}} = \frac{2}{3} \cdot \frac{I_{Friday}}{I_{start\ date}} + \frac{1}{3} \cdot \frac{I_{Monday}}{I_{start\ date}}$$

$$\frac{I_{Sunday}}{I_{start\ date}} = \frac{1}{3} \cdot \frac{I_{Friday}}{I_{start\ date}} + \frac{2}{3} \cdot \frac{I_{Monday}}{I_{start\ date}}$$

The logic can be extended to other situations where an interest calculation ends on a non-business day. For example, if the Monday is a holiday and the next business day is a Tuesday, then one would adjust accordingly:

$$\frac{I_{Saturday}}{I_{start\ date}} = \frac{3}{4} \cdot \frac{I_{Friday}}{I_{start\ date}} + \frac{1}{4} \cdot \frac{I_{Tuesday}}{I_{start\ date}}$$

$$\frac{I_{Sunday}}{I_{start\ date}} = \frac{2}{4} \cdot \frac{I_{Friday}}{I_{start\ date}} + \frac{2}{4} \cdot \frac{I_{Tuesday}}{I_{start\ date}}$$

$$\frac{I_{Monday}}{I_{start\ date}} = \frac{1}{4} \cdot \frac{I_{Friday}}{I_{start\ date}} + \frac{3}{4} \cdot \frac{I_{Tuesday}}{I_{start\ date}}$$

Or, if the interest calculation ends on a holiday in the middle of the week, for example on a Thursday, then interest would be calculated as:

$$\frac{I_{Thursday}}{I_{start\ date}} = \frac{1}{2} \cdot \frac{I_{Wednesday}}{I_{start\ date}} + \frac{1}{2} \cdot \frac{I_{Friday}}{I_{start\ date}}$$

For interest *beginning* on a non-business day, the starting value of the Index would be calculated as an arithmetic weighted average of the SOFR Index values before and after the non-business day. The calculation of compound interest over the period can be based on an interpolation of the Index

values, but in a slightly different way. For example, to calculate compound interest starting on a Saturday or Sunday of a standard weekend, where the preceding Friday and succeeding Monday are both business days, the interpolated values would be:

$$\frac{I_{end\ date}}{I_{Saturday}} = \frac{2}{3} \cdot \frac{I_{end\ date}}{I_{Friday}} + \frac{1}{3} \cdot \frac{I_{end\ date}}{I_{Monday}}$$

$$\frac{I_{end\ date}}{I_{Saturday}} = \frac{2}{3} \cdot \frac{I_{end\ date}}{I_{Friday}} + \frac{1}{3} \cdot \frac{I_{end\ date}}{I_{Monday}}$$

If the succeeding Monday is also a holiday, then the interpolated values would be:

$$\frac{I_{end\ date}}{I_{Saturday}} = \frac{3}{4} \cdot \frac{I_{end\ date}}{I_{Friday}} + \frac{1}{4} \cdot \frac{I_{end\ date}}{I_{Tuesday}}$$

$$\frac{I_{end\ date}}{I_{Sunday}} = \frac{2}{4} \cdot \frac{I_{end\ date}}{I_{Friday}} + \frac{2}{4} \cdot \frac{I_{end\ date}}{I_{Tuesday}}$$

$$\frac{I_{end\ date}}{I_{Monday}} = \frac{1}{4} \cdot \frac{I_{end\ date}}{I_{Friday}} + \frac{3}{4} \cdot \frac{I_{end\ date}}{I_{Tuesday}}$$

And if the interest calculation started on a Thursday holiday in the middle of the week:

$$\frac{I_{end\ date}}{I_{Thursday}} = \frac{1}{2} \cdot \frac{I_{end\ date}}{I_{Wednesday}} + \frac{3}{2} \cdot \frac{I_{end\ date}}{I_{Friday}}$$

Example:

Here we assume that April 1 is a Monday and that March 29 is a Friday. The Index on April 1 is:

$$I_{Apr\ 1} = I_{Mar\ 29}(1 + \frac{3 * SOFR_{Mar\ 29}}{360})$$

To calculate compound interest starting on February 28 and ending on March 30, one would interpolate the SOFR Index in this way:

$$\frac{I_{Mar\ 30}}{I_{Feb\ 28}} = \frac{2}{3} \cdot \frac{I_{Mar\ 29}}{I_{Feb\ 28}} + \frac{1}{3} \cdot \frac{I_{Apr\ 1}}{I_{Feb\ 28}}$$

And to calculate compound interest starting on March 31 and ending on April 30, one would interpolate in the following manner:

$$\frac{I_{Apr\ 30}}{I_{Mar\ 31}} = \frac{1}{3} \cdot \frac{I_{Apr\ 30}}{I_{Mar\ 29}} + \frac{2}{3} \cdot \frac{I_{Apr\ 30}}{I_{Apr\ 1}}$$

For completeness, to calculate interest starting on Saturday March 2 and ending on Sunday March 31, one would interpolate using the business days that covered both the start and end dates:

$$\frac{I_{Mar\ 31}}{I_{Mar\ 2}} = \frac{2}{3} \cdot \frac{\left(\frac{1}{3} I_{Mar\ 29} + \frac{2}{3} I_{Apr\ 1}\right)}{I_{Mar\ 1}} + \frac{1}{3} \cdot \frac{\left(\frac{1}{3} I_{Mar\ 29} + \frac{2}{3} I_{Apr\ 1}\right)}{I_{Mar\ 4}}$$

Appendix 3: Other Potential Lookback Conventions

The Business Loans Working Group discussed several variants of the observation shift that could avoid some of the problems described herein, although they ultimately did not recommend them for syndicated loans. For completeness, we will briefly outline these variants in this appendix. The conventions considered were an interest-period weighted shift, a simple-imputed shift, and a compound-imputed shift.

Interest-Period Weighted Shift

As discussed above, the effective SOFR rate is used to calculate daily accruals. Without a lookback, the effective rate is

$$i_t = \frac{r_t \times n_t}{N}$$

and with a *k*-day lookback but no observation shift, the effective rate is

$$i_t = \frac{r_{t-k} \times n_t}{N}$$

As discussed further in Appendix 3, with no lookback or a lookback without observation shift, the unannualized cumulative compound rate of interest is

$$UCR_t = \left[\prod_{b=1}^{t}(1 + i_t) - 1\right]$$

and the equations and analysis in Appendix 3 can be used as they are, substituting whichever form of effective rate is appropriate. In both those cases, compounding is over the days in the interest period (which is defined in Appendix 3 as d_c, the sum of the n_t)

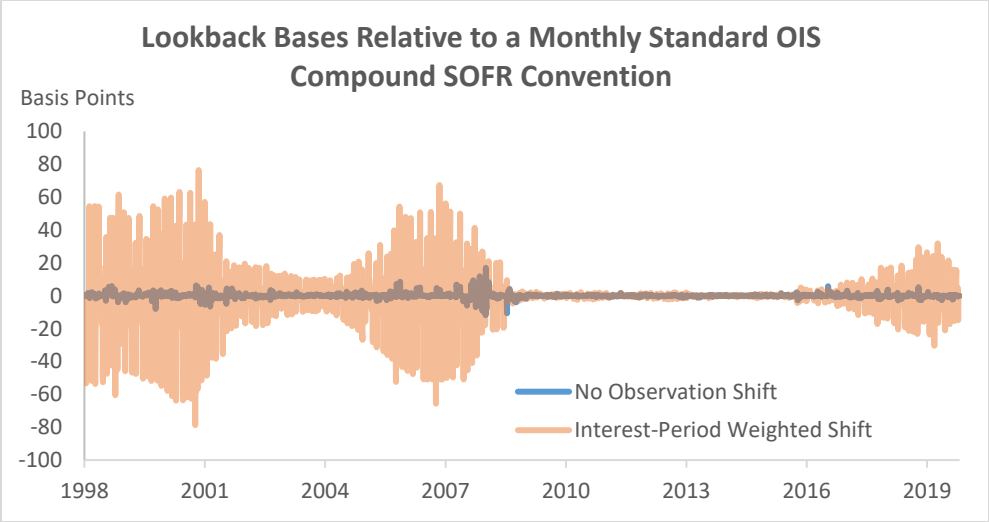
But using a lookback with an observation shift, the effective rate is

$$i_t = \frac{r_{t-k} \times n_{t-k}}{N}$$

and compounding is over the number of days in the observation period (which we will call d_{oc}). As noted, this can lead to a discrepancy between the number of days that interest is charged for and the number of days that the loan is outstanding or held. To correct this, an interest-period weighted shift re-weights the compounded averages to reflect the number of days in the interest period:

$$\left[\prod_{b=1}^{t}(1 + i_t) - 1\right] \cdot \frac{d_{ct}}{do_{ct}}$$

This equation is implicitly used to calculate coupons in some compound SOFR FRNs with lookbacks and observation shifts that specify an interest rate based ISDA's annualized compound SOFR formula and then apply the annualized rate to the number of calendar days in the interest period. The is adjusted lookback does accrue interest for the correct number of days in the interest period; however, compared to a lookback without observation shift, it can have substantial short-term basis relative to a standard SOFR OIS swap, as shown below.[19]



For FRNs, these choices matter less, because principal is constant and so any differences between whether, for example, one interest period has 89 days and another has 91, will tend to average out quickly. In a loan that can be repaid on held for only a short period of time, the calculations may not average out, although a borrower (or lenders) may not place much importance on hedging a loan that they could quickly repay or sell.

Another consideration that led the ARRC not to recommend an interest-period weighted shift is that it can be difficult to implement a daily floor under this convention. Daily accruals may in some circumstances be negative even if SOFR rates are positive or floored. The spreadsheet ARRC BWLG Examples - Other Lookback Options.xlsx demonstrates how to calculate an interest-period weighted shift, and also provides an example of a negative daily accrual under this convention.

Simple-Imputed Shift

The problems with using an observation shift in the syndicated loan market arise when the number of calendar days between two observation dates are different than the number of calendar days between the corresponding interest dates. With a 5-day lookback, this would only occur around holidays. One way around this problem is to impute (or "fill-in") rates for those holiday dates. The

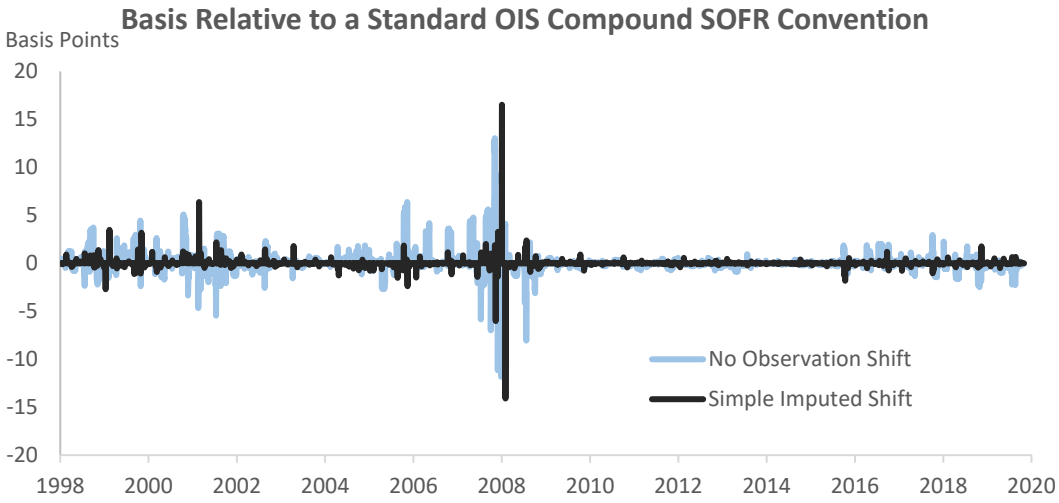
[19] The size of the basis may be surprising, but it reflects the fact that OIS swaps do not have a lookback and that over any given monthly period, the number of days in the observation period can differ from the number of days in the interest period by 3-4 days. For example, one might be 28 days and the other 31 days, which is roughly a 10 percent difference. When rates are high, a 10 percent difference can translate in to 50 basis points or more.

most straightforward way to do this is to apply the rate observed for the date immediately preceding the holiday.

Shift with Simple Imputation (holidays imputed)					
Interest Date (t)	Observation Date (t-5)	Relevant SOFR Print (r_{t-5})	# days rate applies (n_{t-5})	SOFR Effective Rate	SOFR Cumulative Compounded Effective Rate
Mon, July 1, 2019	Mon, June 24, 2019	2.39%	1	0.00664%	0.00664%
Tue, July 2, 2019	Tue, June 25, 2019	2.41%	1	0.00669%	0.01333%
Wed, July 3, 2019	Wed, June 26, 2019	2.43%	1	0.00675%	0.02008%
Thu, July 4, 2019	Thu, June 27, 2019	2.42%	1	0.00672%	0.02681%
Fri, July 5, 2019	Fri, June 28, 2019	2.50%	3	0.02083%	0.04765%
Mon, July 8, 2019	Mon, July 1, 2019	2.42%	1	0.00672%	0.05437%
Tue, July 9, 2019	Tue, July 2, 2019	2.51%	1	0.00697%	0.06135%
Wed, July 10, 2019	Wed, July 3, 2019	2.56%	1	0.00711%	0.06846%
Thu, July 11, 2019	Wed, July 3, 2019	2.56%	1	0.00711%	0.07558%
Fri, July 12, 2019	Fri, July 5, 2019	2.59%	3	0.02158%	0.09718%

The spreadsheet ARRC BWLG Examples - Other Lookback Options.xlsx also demonstrates how to calculate a simple imputed shift. In the screen shot of the spreadsheet shown above, this convention would require calculating interest for July 4 (the calculation itself could take place on July 5, but interest would be compounded separately for July 3 and July 4) using a 5-day lookback to June 27, and in calculating interest for July 11, it would impute a rate for July 4 by using the July 3 rate. With these two rates filled in, the number of days in the observation period for a 5-day lookback would equal the number of days in the interest period.

While this convention does have somewhat less basis relative to a standard SOFR OIS swap than a lookback without observation shift, the differences are very slight – typically less than a basis point. At the same time, implementing this would require nontrivial changes to vendor and lender systems, and the modest improvement in basis did not seem sufficient to warrant such changes.



Compound-Imputed Calendar Shift

This would be essentially the same method as a simple-imputed shift, but rather than taking the last day's rate (which is akin to a simple interest concept) this convention would impute an implied daily compound rate based on the rate from the previous business day. To do this, if the rate on the previous business day before a holiday was *r* and there were *n* calendar days until the next business day, then the imputed daily compounded rate would be

$$\bar{r} = (1 + n \times r)^{\frac{1}{n}} - 1$$

This convention has slightly less basis than a lookback without observation shift relative to a standard OIS swap, but as with the simple-imputed shift, the reduction in basis is slight and adopting the convention would require nontrivial changes to vendor and lender systems. Additionally, BWLG members believed that it would be difficult to explain how the imputed rate had been calculated.

Appendix 4: Methods for Calculating Daily Accrual in an Interest-Period Weighted Observation Shift

ISDA's Compound SOFR formula is based on the following annualized rate calculation:

(A) $\left[\prod_{b=1}^{T}\left(1+\frac{r_b \times n_b}{N}\right)-1\right]\frac{N}{d_c}$

Where d_c is the number of calendar days in the *interest period* $(d_c = \sum_{b=1}^{T} n_b)$

When there is an observation shift, the number of days in the observation period can differ from the number of days in the corresponding interest period. The equivalent of (1) for a framework with an observation shift is:

(B) $\left[\prod_{b=1}^{T}\left(1+\frac{r_{b-k} \times n_{b-k}}{N}\right)-1\right]\frac{N}{d_o}$

Where d_o is the number of calendar days in the *observation period.* $(d_o = \sum_{b=1}^{T} n_{b-k})$.

However there is a question as to how many days equation (B) above would be applied when calculating the coupon payment and calculating daily interest accruals. There a few possible options:

(1) One simple option is to apply interest for the number of days in the observation period associated with the coupon or interest accrual calculation. For an FRN, this would be a workable solution with the understanding that FRN's are long-lived securities and that any difference between the number of days in a given interest period and its associated observation period will average out over time. FRNs trade clean, and secondary trading could take in to account any expectation that the next coupon would be higher or lower based on the difference between observation and interest days. Note that calculating interest based on the number of days in the accrual period and paying 2 business days later is not conceptually different from the payment delay methodology; the principal difference is the start date for the first period where r_i is observed.

(2) A second option, which seems to be widely used in FRNs, is to apply interest based on an equation like (B) above, defining an annualized rate of interest and applying it for the number of days in the interest period associated with the coupon or interest accrual calculation (this variant was labeled an *interest-period weighted shift* in the ARRC technical appendices on loan conventions).

In this convention the coupon payment would be

(2) $\left\{\left[\prod_{b=1}^{T}\left(1+\frac{r_{b-k} \times n_{b-k}}{N}\right)-1\right]\frac{N}{d_o}\right\}\frac{d_c}{N}$

So long as rates are positive, the coupon payment will be positive, but depending on how they are calculated, certain daily accruals may not be positive. Writing out the number of interest days, observation days, and compounding terms up to some date t:

$$d_{ct} = \sum_{b=1}^{t} n_b$$

$$d_{ot} = \sum_{b=1}^{t} n_{b-k}$$

$$\Pi_t = \prod_{b=1}^{t}\left(1 + \frac{r_{b-k} \times n_{b-k}}{N}\right) - 1$$

Accumulated interest up to day *t* have been calculated in this convention as $\Pi_t \frac{d_{ct}}{d_{ot}}$ and daily accruals have been calculated as

$$\Pi_t \frac{d_{ct}}{d_{ot}} - \Pi_{t-1} \frac{d_{ct-1}}{d_{ot-1}}$$

However, daily accruals based on this formulation can be negative if rates are low (although still nonnegative) on business days where the number of observation days jumps up relative to the number of interest days.[20]

(3) There is a *third* alternative to calculate accruals that would work for FRNs (or another security that was long-lived and not paid down before the end of an interest period) and avoid the possibility of negative daily accruals with positive rates.

In this formulation, accumulated interest up to day *t* would be calculated based on the share of interest days up to day *t* relative to the number of observation days in the interest period:

$$\Pi_t \frac{d_{ct}}{d_o}$$

Daily interest accrual in this formulation would be

$$\Pi_t \frac{d_{ct}}{d_o} - \Pi_{t-1} \frac{d_{ct-1}}{d_o}$$

And this would always be positive if rates are positive and the daily accruals would accumulate to the ultimate coupon payment.

The term $\frac{d_{ct}}{d_o}$ can also be written as $\frac{d_{ct}}{d_c}\frac{d_c}{d_o}$ where the first term is the share of interest days over the full coupon period that have occurred up to time *t* and the second term is the final weighting function determining the coupon payment based on the daycount convention and the annualized rate of interest. With this view, the same convention could be used to accrue interest using a 30/360 coupon convention rather than an Act/360 or Act/365 convention, by using $\frac{d_{ct}}{d_c}\frac{30}{d_o}$ instead of $\frac{d_{ct}}{d_o}$ above.

[20] The equation can be rewritten as $\frac{r_{t-k} \times n_{t-k}}{N}\frac{d_{ct}}{d_{ot}}\Pi_{t-1} + \left(\frac{d_{ct}}{d_{ot}} - \frac{d_{ct-1}}{d_{ot-1}}\right)\Pi_{t-1}$ and] the second term will be negative if the ratio of interest to observation days on day t is less than the ratio for t-1, and this term can be larger than the first term (which will be nonnegative) if rates are low.

(4) To be complete, there is a fourth alternative: accrued interest could also be calculated based on

$\Pi_t \frac{d_c}{d_o}$ rather than $\Pi_t \frac{d_{ct}}{d_o}$.

The formulation of accrual laid out in alternative (3) above recognizes that, even if the daily rate was zero, the amount of interest charged to the issuer will increase if rates had been positive earlier in the coupon period, because the issuer will be charged the average rate of interest applied to the number of interest days, but either option (3) or (4) will accrue to the correct final coupon payment and reflect daily interest rate movements based on the observation shift.

Market participants (and accounting experts) would need to determine if accrual formulations (1), (3) or (4) above were preferable to the interest-period weighted shift. Arguably a convention that produces positive daily accruals when daily rates are positive makes some intuitive economic sense. We note that these conventions would work for an FRN but not necessarily for a loan that can be repaid or paid down at any point in the interest period at the discretion of the borrower, essentially because there is no guarantee that the loan will remain in effect for the entire interest period and that the terms d_c and d_o have any bearing on the amount of interest that the borrower should owe if it is repaid early.

Appendix 4: Key Provisions for Daily Simple SOFR Loan Facility with Lookback (No Observation Shift)[1]

Interest Amount: For SOFR loans, the amount of interest accrued and payable on the loans for any day will be equal to the product of (i) the outstanding principal amount of the loans on such day multiplied by (ii) (a) the Rate of Interest for such day divided by (b) 360.

Rate of Interest: The Benchmark plus the Applicable Margin.

Benchmark: For SOFR loans, the benchmark is Daily Simple SOFR.

Applicable Margin: The margin is [plus]/[minus] ___ basis points per annum.

Interest Payment Dates: The last business day of each [March, June, September and December][calendar month] and the Maturity Date.

U.S. Government Securities Business Day: Any day except for (i) a Saturday, (ii) a Sunday or (iii) a day on which the Securities Industry and Financial Markets Association recommends that the fixed income departments of its members be closed for the entire day for purposes of trading in United States government securities.

Day count convention: Actual/360

Floor: ___%.[2]

SOFR: means, with respect to any U.S. Government Securities Business Day, a rate per annum equal to the secured overnight financing rate for such U.S. Government Securities Business Day, as such rate appears on the SOFR Administrator's Website on the immediately succeeding U.S. Government Securities Business Day.[3]

where:

> **"SOFR Administrator"** means the Federal Reserve Bank of New York (or a successor administrator of the secured overnight financing rate).
>
> **"SOFR Administrator's Website"** means the website of the Federal Reserve Bank of New York, currently at http://www.newyorkfed.org, or any successor source for the secured overnight financing rate identified as such by the SOFR Administrator from time to time.

Daily Simple SOFR: means, for any day (a "**SOFR Interest Day**"), an interest rate per annum equal to the greater of (a) SOFR for the day that is [five] U.S. Government Securities Business Days[4] prior to (i) if such SOFR Interest Day is a U.S. Government Securities Business Day, such SOFR Interest Day or (ii) if such SOFR Interest Day is not a U.S. Government Securities Business Day, the U.S. Government Securities Business Day immediately preceding such SOFR Interest Day and (b) the Floor. Any change in

[1] A business day lookback with no observation shift is the recommended convention for business loans. The lookback looks a certain number of business days backward to ascertain SOFR for a given day.
[2] Business loans routinely include zero or non-zero LIBOR floors.
[3] For any day it is published (at approx. 8:00 a.m.), the published SOFR can be revised until approximately 2:30 p.m. of the day it is published. Parties may want to indicate the time of SOFR "as it appears".
[4] The length of the lookback to be synced with the timings of Borrowing Requests and Prepayment Notices.

Daily Simple SOFR due to a change in SOFR shall be effective from and including the effective date of such change in SOFR without notice to the borrower.

Borrowing Requests: Notice of each SOFR borrowing must be received by the Administrative Agent or Lender, as applicable, not later than [11:00 a.m.] ([New York City time]) [five][5] business days prior to the date of the requested borrowing.

Prepayment Notices: Notice of a voluntary prepayment must be received by the Administrative Agent or Lender, as applicable, not later than [11:00 a.m.] ([New York City time]) [five][6] business days before the date of prepayment.

[5] See footnote 4 above.
[6] See footnote 4 above.

Appendix 5: Key Provisions for a SOFR in Advance Loan Facility

Interest Amount: For SOFR loans, the amount of interest accrued and payable on the loans for any Interest Period will be equal to the product of (i) the outstanding principal amount of the loans multiplied by (ii) (a) the Rate of Interest for such Interest Period divided by (b) 360.

Rate of Interest: The Benchmark plus the Applicable Margin.

Benchmark: The Benchmark is 30-Day Average SOFR.

Applicable Margin: The margin is [plus]/[minus] ___ basis points per annum.

Interest Period: For SOFR loans, the period commencing on the date of such loan and ending on the numerically corresponding day in the calendar month that is [one month][three months] thereafter.

Interest Payment Dates: The last day of each Interest Period and the Maturity Date.

Interest Determination Date: The date [two] U.S. Government Securities Business Days prior to (i) if such day is a U.S. Government Securities Business Day, the commencement of the Interest Period or (ii) if such day is not a U.S. Government Securities Business Day, the U.S. Government Securities Business Day immediately preceding the commencement of the Interest Period.

U.S. Government Securities Business Day: Any day except for (i) a Saturday, (ii) a Sunday or (iii) a day on which the Securities Industry and Financial Markets Association recommends that the fixed income departments of its members be closed for the entire day for purposes of trading in United States government securities.

Business Day Convention: Modified Following; Adjusted. (i) If any Interest Period would end on a day other than a business day, such Interest Period shall be extended to the next succeeding business day unless such next succeeding business day would fall in the next calendar month, in which case such Interest Period shall end on the next preceding business day, (ii) any Interest Period that commences on the last business day of a calendar month (or on a day for which there is no numerically corresponding day in the last calendar month of such Interest Period) shall end on the last business day of the last calendar month of such Interest Period and (iii) no Interest Period shall extend beyond the Maturity Date.

Day count convention: Actual/360

Floor: ___%.[1]

30-Day Average SOFR: means, for any Interest Period, the greater of: (a) the 30-Day Average SOFR published on the SOFR Administrator's Website as of the applicable Interest Determination Date and (b) the Floor.

where:

[1] Business loans routinely include zero or non-zero LIBOR floors.

"SOFR Administrator" means the Federal Reserve Bank of New York (or a successor administrator of the secured overnight financing rate).

"SOFR Administrator's Website" means the website of the Federal Reserve Bank of New York, currently at http://www.newyorkfed.org, or any successor source for the secured overnight financing rate identified as such by the SOFR Administrator from time to time.

Appendix 6: Key Provisions for an Interest-Adjusted SOFR in Advance Loan Facility

Interest Amount: For SOFR loans, the amount of interest accrued and payable on the loans for any Interest Period will be equal to the product of (i) the outstanding principal amount of the loans multiplied by (ii) (a) the Rate of Interest for such Interest Period divided by (b) 360.

Rate of Interest: The Benchmark plus the Applicable Margin.

Benchmark: The Benchmark is Interest-Adjusted 30-Day SOFR.

Applicable Margin: The margin is [plus]/[minus] ___ basis points per annum.

Interest Period: For SOFR loans, the period commencing on the date of such loan and ending on the numerically corresponding day in the calendar month that is [one month][three months] thereafter.

Interest Payment Dates: The last day of each Interest Period and the Maturity Date.

Interest Determination Date: The date [two] U.S. Government Securities Business Days prior to (i) if such day is a U.S. Government Securities Business Day, the commencement of the Interest Period or (ii) if such day is not a U.S. Government Securities Business Day, the U.S. Government Securities Business Day immediately preceding the commencement of the Interest Period.

U.S. Government Securities Business Day: Any day except for (i) a Saturday, (ii) a Sunday or (iii) a day on which the Securities Industry and Financial Markets Association recommends that the fixed income departments of its members be closed for the entire day for purposes of trading in United States government securities.

Business Day Convention: Modified Following; Adjusted. (i) If any Interest Period would end on a day other than a business day, such Interest Period shall be extended to the next succeeding business day unless such next succeeding business day would fall in the next calendar month, in which case such Interest Period shall end on the next preceding business day, (ii) any Interest Period that commences on the last business day of a calendar month (or on a day for which there is no numerically corresponding day in the last calendar month of such Interest Period) shall end on the last business day of the last calendar month of such Interest Period and (iii) no Interest Period shall extend beyond the Maturity Date.

Day count convention: Actual/360

Floor: ___%.[1]

30-Day Average SOFR: means the 30-Day Average SOFR published on the SOFR Administrator's Website.

where:

[1] Business loans routinely include zero or non-zero LIBOR floors.

"SOFR Administrator" means the Federal Reserve Bank of New York (or a successor administrator of the secured overnight financing rate).

"SOFR Administrator's Website" means the website of the Federal Reserve Bank of New York, currently at http://www.newyorkfed.org, or any successor source for the secured overnight financing rate identified as such by the SOFR Administrator from time to time.

Interest-Adjusted 30-Day SOFR: means, for any Interest Period,

(a) for the first Interest Determination Date of a SOFR loan, the greater of: (i) the 30-Day Average SOFR and (ii) the Floor, and

(b) for any subsequent Interest Determination Date for that SOFR loan, the greater of: (i) the sum of (A) the 30-Day Average SOFR for the current interest determination date and (B) the difference between: (x) the 30-Day Average SOFR for the current interest determination date and (y) the 30-Day Average SOFR for the immediately preceding interest determination date and (ii) the Floor.